INSTITUTE *of* STATISTICS
Oxford

Bulletin Vol. 16 *Nov. & Dec., 1954* *Nos. 11 & 12*

LABOUR PRODUCTIVITY AND SIZE OF ESTABLISHMENT[1]

This article examines the relationship between labour productivity and size of establishment in the light of the summary data contained in the U.S.A. Censuses of Manufactures. In particular, it explores the statistical paradox of the conflicting results that emerge according to the classification methods adopted : the cause of this paradox is explained and suggestions made for establishing correct inferences about the variations of labour productivity with size of establishment.

I

It is a widely held belief that labour productivity tends in general to increase with increases in the size of establishment; and this generalisation seems to be strongly supported by the summary tables of the U.S.A. Censuses of Manufactures. Taking as indicators of labour productivity, value of product per wage earner and also value added per wage earner we have the results given in Table I.[2] With only two exceptions, both indicators in Table I

TABLE I. *Labour productivity and size of establishment, U.S.A. 1914–1939*[1,2]

Value of product $*000*	*1914*		*1919*		*1929*		*1939*	
	(*a*) $	(*b*) $	(*a*) $	(*b*) $	(*a*) $	(*b*) $	(*a*) $	(*b*) $
less than 5 ...	1,800	1,171	3,647	2,328	—	—	—	—
5—	2,111	1,183	3,787	2,162	3,801	2,361	3,591	2,088
20—	2,551	1,239	4,501	2,203	4,739	2,800	4,135	2,290
50—					5,542	3,180	4,649	2,454
100—	2,919	1,295	5,213	2,414	5,865	3,101	5,103	2,520
250—					6,102	3,086	5,513	2,583
500—			5,802	2,586	6,504	3,157	5,886	2,713
1000—	4,763	1,653	8,178	3,017	7,088	3,356	6,354	2,923
2500—					7,802	3,671	7,652	3,496
5000 and over ...					11,790	4,577	11,247	4,180
All establishments	3,446	1,404	6,862	2,752	7,969	3,607	7,208	3,130

Sources : The figures are computed from the *Fourteenth Census of the United States*, Vol. VIII, Table 21 ; *Fifteenth Census of the United States*, Manufactures, Vol. I, Table 2 ; and 1939 *Census of Manufactures*, Vol. I, Chapter IV, Table 6.

[1] The establishments are classified according to the total value of their product, and the figures cover the establishments in all the trades included in each census.

[2] (*a*) Value of product per wage earner.
(*b*) Value added per wage earner.

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[1] This paper was completed as part of a wider study of Cost-Output Relationships, which is being assisted by a research grant from the Houblon-Norman Fund.

[2] Our primary concern is not with the relative merits of particular indicators of labour productivity, but rather with the conflicting results that can emerge from census material for any given indicator.

record a steady rise in labour productivity with size of establishment in all four years.

In 1939, the census establishments were also classified according to the number of wage earners, and in 1947 the only basis of classification employed was number of employees. The results for these two years are given in Table II. These results provide a striking contrast to those of Table I. For 1939 both indicators decline sharply through the first six size classes to a value about 65 per cent of the initial maximum value and thereafter increase, except for product per wage earner which falls again in the final size group. For 1947 the pattern is similar, but less violent: the initial fall only covers three size groups and is only about 20 per cent of the commencing figure: thereafter a rise takes place and then a flattening out and decline at the top end of the size scale. Thus two markedly different patterns emerge for the relationship of labour productivity to size of establishment, depending upon whether the establishments are classified by value of product or number of wage earners.

TABLE II

Labour productivity and size of establishment, U.S.A., 1939 and 1947

	1939		*1947*	
No. of wage earners per establishment	*Product per wage earner* $	*Value added per wage earner* $	*No. of employees*	*Value added per employee* $
1—5	10,074	4,371	1—4	5,624
6—20	7,552	3,362	5—9	4,655
			10—19	4,628
21—50	6,772	3,027	20—49	4,864
51—100	6,482	2,940	50—99	5,070
101—250	6,437	2,902	100—249	5,293
251—500	6,421	2,838	250—499	5,416
501—1,000	7,220	3,044	500—999	5,418
1,001—2,500	8,456	3,340	1000—2499	5,414
2,501 & over	7,918	3,374	2500 & over	5,051
All establishments	7,208	3,130	All establishments	5,207

Sources: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Table 1; U.S.A. *Census of Manufactures*, 1947, Vol. I, Chapter III, Table 1.

This problem is examined briefly in Steindl's *Maturity and Stagnation in American Capitalism*, pages 18–24, but, although his results are mathematically correct, they are economically unenlightening and lead him to accept the classification by value of product, which, as we shall see in Section II, may be a rather dangerous procedure.[1]

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[1] Steindl's approach is to pose the question: given a *positive* correlation between the log of value of product per wage earner and log of value of product per establishment, under what conditions is it possible to have a *negative* correlation between log of value of product per wage earner and log of number of wage earners per establishment? Letting r_1 denote the first correlation and b_1 the slope of the linear regression of log of value of product per wage

[Footnote continued on following page

II

Columns (2), (3), (6) and (7) of Table III show that value of product and number of wage earners per establishment both increase steadily up the size groups on each basis of classification, but product per wage earner shows two markedly different patterns. This suggests that the cause of the paradox must be related to the two different bases of classification.

TABLE III

Comparison of the two bases of classification, 1939

Classificatiion by value of product				*Classification by number of wage earners*			
Value of product ($000) (1)	*Value of product per establishment* $ (2)	*Wage earners per establishment* (3)	*Value of product per wage earner* ($) (4)	*No. of wage earners per establishment* (5)	*Value of product per establishment* $ (6)	*Wage earners per establishment* (7)	*Value of product per wage earner* ($) (8)
5—	11,235	3.13	3,591	1–5	26,939	2.67	10,074
20—	32,167	7.78	4,135	6–20	83,615	11.07	7,552
50—	71,066	15.29	4,649	21–50	219,042	32.34	6,772
100—	158,628	31.08	5,103	51–100	461,855	71.25	6,482
250—	354,120	64.23	5,513	101–250	1,002,245	155.70	6,437
500—	701,923	119.25	5,886	251–500	2,230,629	347.38	6,421
1000—	1,524,304	240.35	6,354	501–1000	4,946,892	685.15	7,220
2500—	3,437,122	449.17	7,652	1001–2500	12,499,101	1478.13	8,456
5000 and over	15,017,720	1335.25	11,247	2501 & over	37,094,179	4684.84	7,918

Source: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Tables 1 and 6.

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earner on log of value of product per establishment, the answer is that the negative correlation is possible if

$$r_1^2 < b_1 \text{* (*\textit{op. cit.} pp. 19–20)} \qquad (1)$$

The algebra is impeccable, but the following points should be noted.

1. The problem which Steindl set himself is not, in fact, the problem raised by the 1939 census results. The latter give a smooth upward movement for labour productivity on the one classification and a down-up-down movement on the other: the condition for this result may possibly be less severe than that required to give a *negative linear* regression in the second case. The completely contradictory patterns do, however, appear for some of the industry groups in Section III below.

2. Condition (1) may be written:

$$b_1 b_2 < b_1 \qquad (2)$$

where b_2 is the slope of the linear regression of log of value of product per establishment on log of value of product per wage earner. Cancelling out b_1 which is positive by assumption, leaves

$$b_2 < 1 \qquad (3)$$

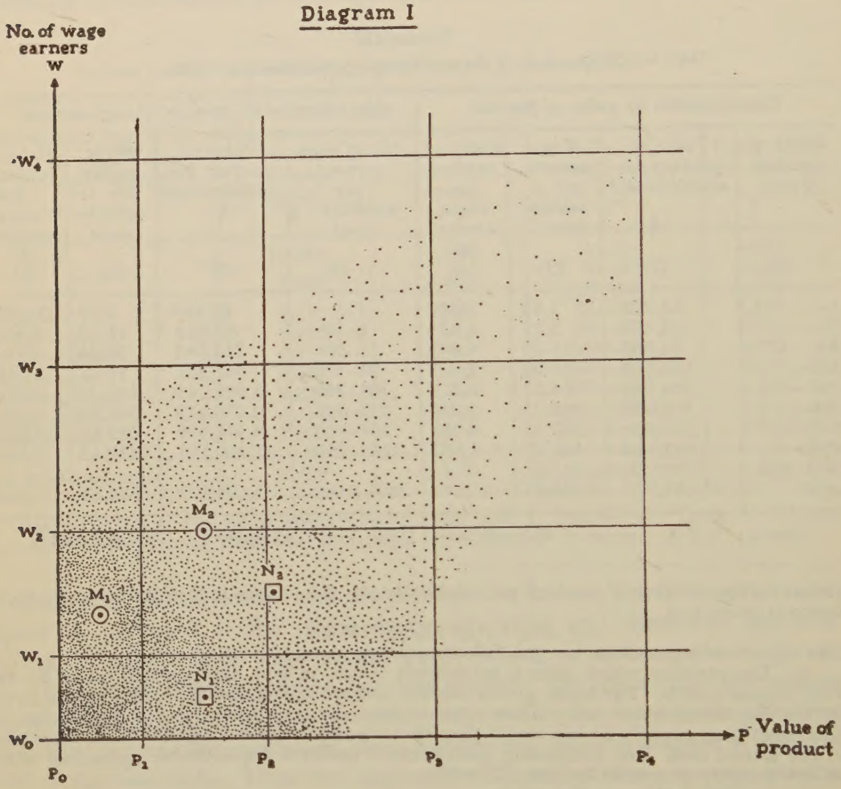
Denoting value of product by P and number of wage earners by W, the appropriate regression may be written

$$\log P = a_2 + b_2 \log\left(\frac{P}{W}\right) \qquad (4)$$

If we have a series of establishments whose P, W co-ordinates satisfy (4) exactly, then it follows that for these establishments an increase of x per cent, say, in $\left(\frac{P}{W}\right)$ is associated with an increase of less than x per cent in P. This can only come about if P and W are *inversely* related, if, in other words, the capital structure of these establishments is such that those which are larger as measured by value of product are smaller when measured by number of wage earners. This seems a strange result, and suggests that the regression relationship (4) may sometimes be rather meaningless. It is, therefore, essential to examine the shape of the scatter about this regression in specific cases, and this is done in the Appendix.

Suppose that we have a scatter diagram on which value of product, (P), is measured on the horizontal scale and number of wage earners, (W), on the vertical, each point on the scatter relating to a single establishment, and suppose further that the range of each variable is marked off in certain intervals as in Diagram I.

Diagram I



Classifying the establishments according to value of product or number of wage earners means splitting the diagram into vertical or horizontal slices respectively and then computing for the establishments in each slice the appropriate statistic such as value of product per wage earner, or value added per wage earner. If there are n establishments in a given group then for that group

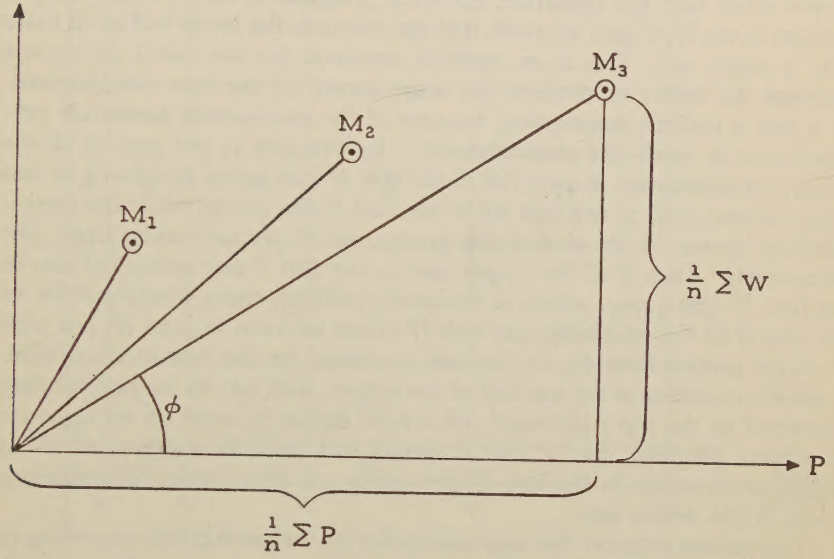
Value of product per establishment $= \Sigma P/n$.

Number of wage earners per establishment $= \Sigma W/n$.

Value of product per worker $= \Sigma P/\Sigma W = \frac{1}{n}\Sigma P \Big/ \frac{1}{n}\Sigma W$

Thus geometrically the value of product per wage earner for the establishments in a group is given by cot ϕ, where ϕ is the angle formed by the P-axis and the straight line joining the origin to the point whose co-ordinates are $(\frac{1}{n}\Sigma P, \frac{1}{n}\Sigma W)$. If product per worker is to *increase* from one group to another, then the angle ϕ must *decrease* from the first group to the second (and *vice versa*). This relationship is illustrated in Diagram II. M_1, M_2, and M_3 are three group means which indicate rising labour productivity, since the angle made with the P-axis decreases as we move from Group 1 to 3.

Diagram II



Referring now to Diagram I we can see how the paradoxical effect arises. Consider the smallest value of product size group from P_0 to P_1: this comprises all establishments, say n' in number, in the first vertical slice of the diagram. For these establishments $\frac{1}{n'}\Sigma P$ will obviously be about midway between P_0 and P_1. But the W values range from W_0 to over W_2 and $\frac{1}{n'}\Sigma W$ will probably lie above W_1. These co-ordinates suffice to define the point M_1 on the diagram. Similarly M_2 is the mean point for all establishments whose P-value lies in the range P_1 to P_2. For the n'' establishments whose W-value lies in the range W_0 to W_1, the P-values range from P_0 to over P_2 and so the mean point $(\frac{1}{n''}\Sigma P, \frac{1}{n''}\Sigma W)$ can be represented by N_1 on the diagram. Similarly N_2 represents the mean point for all establishments in the second W classification, from W_1 to W_2. This type of pattern produces two results:

(*a*) Value of product per worker is much smaller for the establishments in the first P-classification than for those in the first W-classification, since angle $M_1OP >$ angle N_1OP.

(*b*) Value of product per worker increases from the first to the second *P*-group and decreases from the first to the second *W*-group, since angle $M_1OP >$ angle M_2OP, and angle $N_1OP <$ angle N_2OP.

Both these results are found in the data in Table III; product per wage earner in the first *P* classification is $3,591 compared with $10,074 in the first *W* classification, and movement to the second classification gives a positive and negative change respectively.

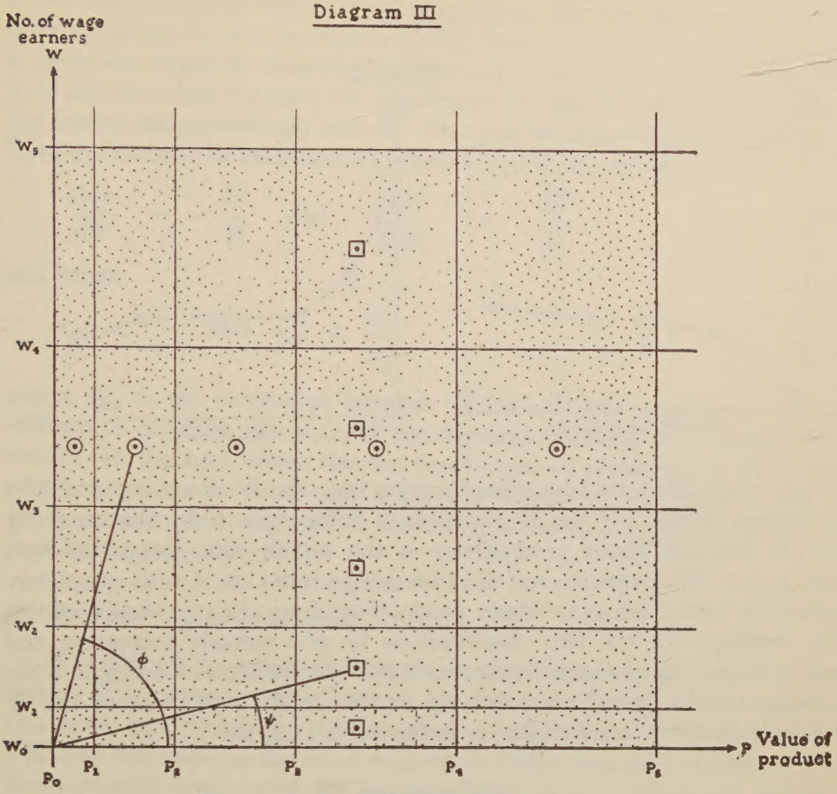
We have postulated nothing so far about the general shape of the *W*, *P* scatter other than the condition, shown in Diagram I, that it is so wide, in relation to the *W*, *P* grid adopted, that the points at the lower end lie in more than a single cell. This is an essential condition for the initial divergence between the values of product per wage earner on the two classifications. It is also a realistic assumption, because of the tremendous numerical predominance of small-size establishments: for example 43 per cent of all the census establishments in 1939 fell in the first *W*-size group (i.e. had 5 or less wage earners), and 33 per cent fell in the first *P*-size group, while the corresponding figures for the second size groups are 28 per cent and 23 per cent respectively. Even if all the 33 per cent in the first *P* size group fell also in the first *W* size group, which is extremely unlikely, there would still be 10 per cent of all the establishments with *W* values between W_0 and W_1 but with *P* values greater than P_1, so that the condition on the scatter is satisfied. A similar condition at the top end of the scatter, with not all the points being contained in the top right-hand cell, would suffice to open up an opposite divergence between the *W* and *P* means and possibly produce a fall in labour productivity in the last *W* size group, an effect which is sometimes found in the census data.

The various patterns that may emerge for labour productivity according to the classification employed on a given set of census data may now be outlined. The distinction between the various cases rests on the initial assumptions which we make about the nature of the *W*,*P* scatter.

CASE A: ZERO CORRELATION IN THE *W*,*P* SCATTER

We assume here that the points of the scatter are fairly evenly spread over the *W*,*P* grid. If ⊙ denotes the mean points $(\frac{1}{n}\Sigma P, \frac{1}{n}\Sigma W)$ for the *P*-size groups and ⊡ denotes the mean points for the *W*-size groups, then the results will be as shown in Diagram III. The *P* classification gives value of product per wage earner $=$ cot ϕ, which *increases* steadily as *P* increases; and the *W* classification gives value of product per wage earner $=$ cot ψ, which *decreases* steadily as *W* increases. Thus, starting with a case in which there is no connection at all between *W* and *P*, one emerges with apparently emphatic indications of a strong relationship, positive or negative, between labour productivity and size, depending upon the method of classification employed.

Diagram III



CASE B. POSITIVE CORRELATION IN THE *W*, *P* SCATTER

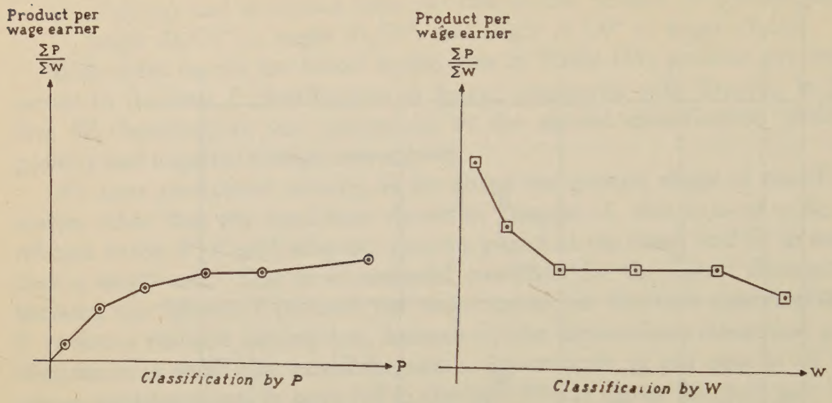
The most probable *W,P* pattern is perhaps one in which the points slope roughly up the diagram from left to right, and it is necessary to distinguish three sub-cases, according to whether there is a tendency towards constant, increasing, or decreasing labour productivity.

(i) *Constant labour productivity*:

If there is a tendency towards constant labour productivity, the points on the *W,P* scatter will be distributed approximately around a straight line through the origin, the slope of the line, referred to the *W*-axis, indicating the average productivity level. Owing to the effect described on pages 342 to 344 above, the pattern that will emerge for labour productivity will probably be as indicated in Diagram IV.

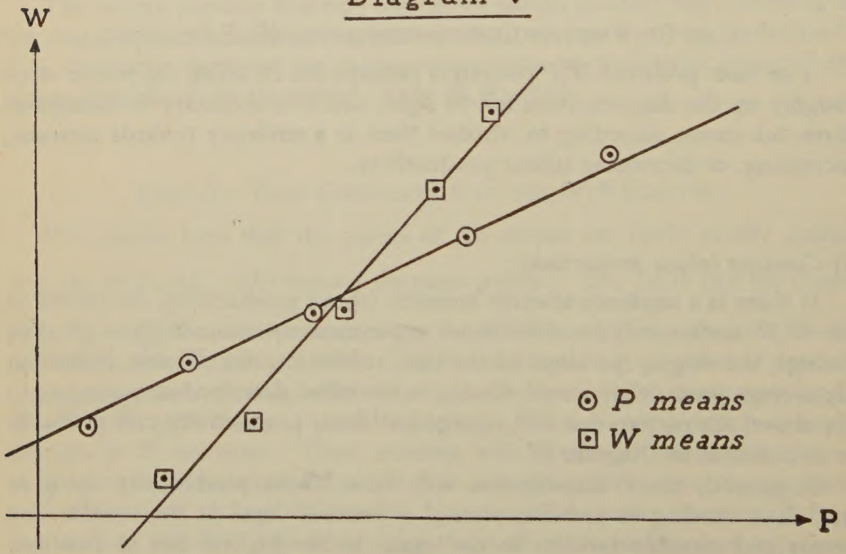
In general, the *P* classification will show labour productivity rising at first, then tending to stabilise around a constant level in the middle size groups and possibly tending to rise again in the highest one or two size

Diagram IV



groups : the pattern for labour productivity from the *W*-classification will be the reverse of this. If the *W,P* scatter is sufficiently wide, the tendency towards constant labour productivity in the middle size groups on each classification will disappear, and this case merges into Case A with completely conflicting indications as to labour productivity from the two classifications. The condition for the two classifications to give steadily increasing and steadily decreasing productivity figures is that the lines from the origin to the *P* means should describe successively smaller angles and those to the *W* means successively larger angles, as indicated in Diagram V :

Diagram V



In the case of approximately linear regressions, this merely requires that the intercepts made by the regressions on the appropriate axes should be positive. It is obvious from the diagram that this may occur even when the regression lines are very close together, the correlation between W and P fairly high and the scatter correspondingly narrow. Using small letters to denote deviations from the means, the condition on positive intercepts may be written

$$\frac{\Sigma wp}{\Sigma p^2} < \frac{\overline{W}}{\overline{P}} \quad \text{and} \quad \frac{\Sigma w^2}{\Sigma wp} > \frac{\overline{W}}{\overline{P}} \tag{5}$$

and hence

$$r_{wp} < \frac{v_p}{v_w} \quad \text{and} \quad r_{wp} < \frac{v_w}{v_p} \tag{6}$$

where r_{wp} is the correlation between W and P, and v denotes the coefficient of variation (the ratio of the standard deviation of a variable to its mean). In the case where the two coefficients of variation are fairly close together, the smaller of the two ratios will still be close to unity so that the W, P scatter can be fairly narrow and still give rise to the contrary indications on labour productivity.

(ii) *Increasing labour productivity*

It is difficult to define unambiguously what sort of pattern on the W, P scatter will correspond to a case of increasing productivity. But there are two main considerations : firstly, points displaying increasing productivity with size will slope up the W, P diagram in such a way as to describe successively smaller angles with the horizontal axis at the origin; and secondly, there will almost always be observations fairly close to the origin in the smallest W and P size groups. Hence we may say that a case of increasing productivity will tend to give a W, P scatter which is curvilinear and concave from below, as shown in Diagram VI. This would give the labour productivity patterns shown in Diagram VII.

The scatter effect already described would be sufficient to give declining labour productivity here over the first three W-groups. If there were more than 5 size groups the range of increasing labour productivity on the W classification would be extended, and there exists also the possibility of a fall in the final size group.

It is also clear from Diagram VI that if we took a number of heterogeneous industries, most of which showed increasing labour productivity, but whose products differed substantially in monetary value, and if we combined this data on a single W, P diagram, then the scatter would approximate that pictured in Case A above, so that the W-classification could show strong signs of decreasing labour productivity over the whole size range, in spite of the separate underlying tendencies towards increasing productivity.

Diagram VI

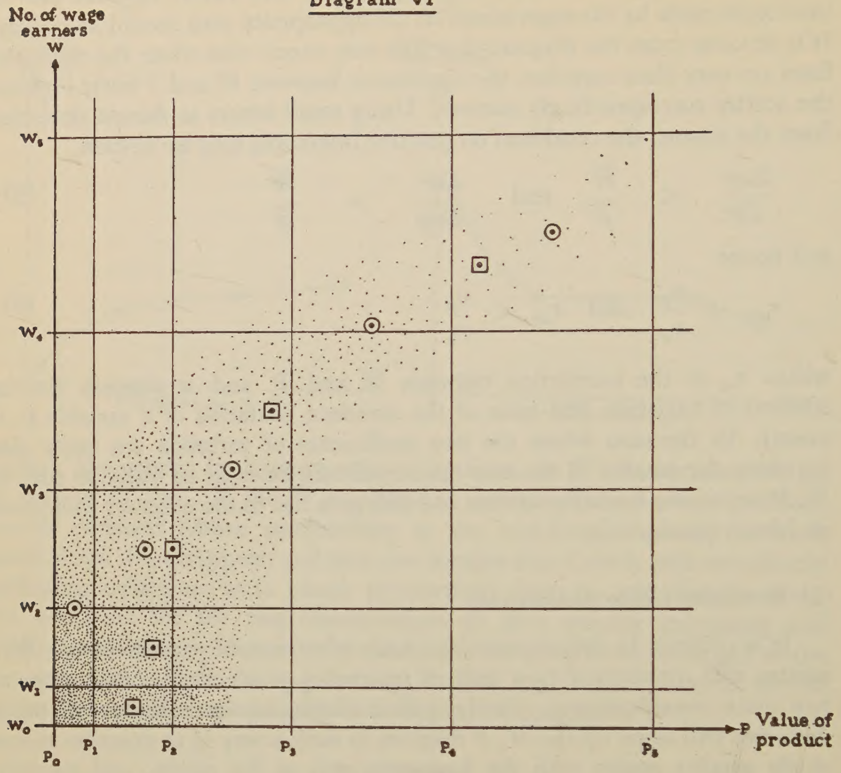
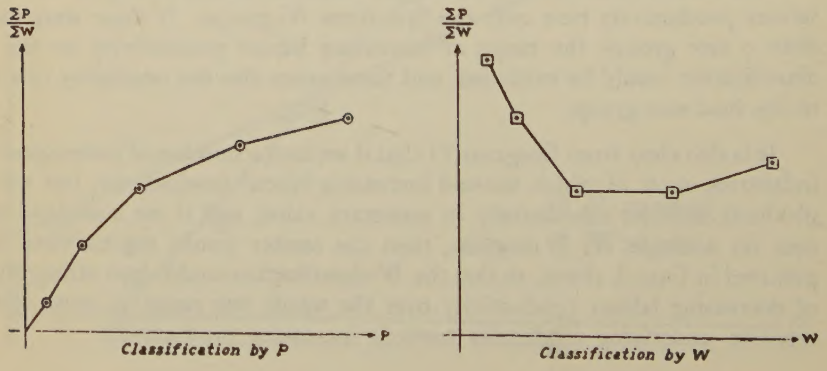


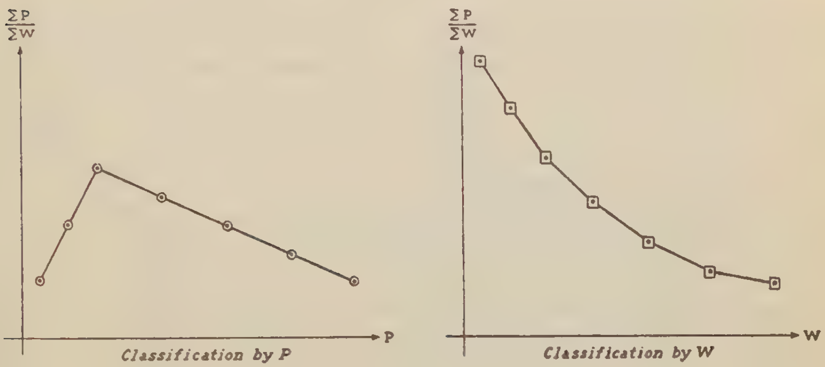
Diagram VII



(iii) *Decreasing labour productivity*

This tendency would be described by a curvilinear scatter, convex from below. In this case the *P*-classification would give a productivity curve increasing at first and then declining, while the *W*-classification would give a productivity curve tending to decline through all size-groups as shown in Diagram VIII.

Diagram VIII



CASE C: NEGATIVE CORRELATION IN THE *W*,*P* SCATTER

This is an unrealistic case, included only for the sake of completeness. If the *W*, *P* scatter has a negative slope, it is easy to see that labour productivity will increase steadily with size on the product classification and decrease steadily on the number of wage earners classification. This case is of no practical importance and it would not give, for example, the result noted in Table III of $\frac{1}{n}\Sigma P$ and $\frac{1}{n}\Sigma W$ being positively related in all the size groups of either classification.

Of the five cases outlined above, three tend to produce a steadily rising labour productivity curve on the *P*-classification, and the other two will almost certainly give at least an initial rise. It seems clear, therefore, that inferences about the relationship of labour productivity to size can only be established by joint consideration of both patterns. Diagrams IV, VII and VIII are probably the crucial patterns and enable us to distinguish between cases of constant, increasing or decreasing labour productivity. If we find instead that the curve on the *P*-classification slopes steadily upwards throughout its length, while that on the *W*-classification slopes steadily downwards then we have an example of Case A, where the *W*, *P* scatter must be very wide and no relationship can be established: this case would result from the merger of data on insufficiently homogeneous industries, or it may merely reflect a great dispersion of productivity within an industry.

III

To test the validity of the above hypotheses and also to see if apparently conflicting patterns still emerge for finer sub-groupings of industry we present in Tables IV and V the value of product per wage earner computed from 20 major industry groupings on each basis of classification for the 1939 U.S.A. Census of Manufactures. The industry groups are:

1. Food and kindred products.
2. Tobacco manufactures.
3. Textile-mill products and other fibre manufactures.
4. Apparel and other finished products made from fabric and similar materials.
5. Lumber and timber basic products.
6. Furniture and finished lumber products.
7. Paper and allied products.
8. Printing, publishing and allied industries.
9. Chemicals and allied products.
10. Products of petroleum and coal.
11. Rubber products.
12. Leather and leather products.
13. Stone, clay, and glass products.

Table IV

Value ($) of product per wage earner, U.S.A. 1939. Classification by value of product

	Size groups (value of product $000)								
Industry group	*A* *5–*	*B* *20–*	*C* *50–*	*D* *100–*	*E* *250–*	*F* *500–*	*G* *1,000*	*H* *2,500–*	*I* *5,000 & over*
1	5,679	7,461	8,855	9,371	10,161	11,297	13,017	14,853	22,776
2	2,409	2,192	2,454	2,812	3,527	2,799	3,502	5,626	27,199
3	1,826	2,371	2,728	2,965	3,194	3,364	3,663	4,010	4,320
4	1,878	2,159	2,765	3,955	5,049	5,819	5,515	6,199	7,486
5	1,577	2,289	2,897	3,110	3,280	3,509	3,846	3,957	*
6	3,331	3,787	3,934	3,977	4,137	4,669	4,830	4,314	6,467
7	3,100	3,425	4,053	4,740	5,928	7,203	8,222	9,017	9,720
8	4,685	5,424	5,885	6,566	7,281	7,400	8,875	9,386	18,778
9	7,398	9,657	10,475	11,246	12,109	13,552	14,712	14,070	12,952
10	5,748	7,785	10,780	12,893	15,920	21,000	22,552	23,370	31,351
11	3,659	4,625	5,213	4,799	4,407	4,926	4,473	6,014	10,340
12	3,131	3,494	3,433	3,640	3,594	3,912	4,304	5,362	5,513
13	3,186	3,644	3,750	4,121	4,065	4,719	6,135	5,498	6,713
14	3,720	4,158	4,283	4,669	4,877	5,062	5,840	7,741	8,431
15	4,564	4,826	5,414	5,805	6,375	7,243	8,138	8,942	17,966
16	4,384	5,269	5,243	5,387	5,660	5,806	6,282	6,501	7,930
17	3,918	4,590	5,110	5,463	5,917	6,383	6,274	6,766	6,570
18	4,008	4,501	4,816	5,214	6,086	6,359	6,394	6,710	10,886
19	3,102	3,272	3,187	3,735	4,029	4,451	5,087	5,249	6,389
20	3,446	3,753	4,005	4,047	4,192	4,754	5,090	7,398	12,339
All industry groups ...	3,591	4,135	4,649	5,103	5,513	5,886	6,354	7,652	11,247

Source: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Table 7.

* Indicates that the figure for this cell has been merged with that for the previous cell to avoid disclosure of information relating to one or two establishments.

14. Iron and steel and their products except machinery.
15. Nonferrous metals and their products.
16. Electrical machinery.
17. Machinery (except electrical).
18. Automobiles and automobile equipment.
19. Transportation equipment except automobiles.
20. Miscellaneous industries.

TABLE V

Value ($) of product per wage earner, U.S.A. 1939. Classification by number of wage earners

Industry group	*Size Groups (No. of Wage Earners)*								
	1 *1–5*	*2* *6–20*	*3* *21–50*	*4* *51–100*	*5* *101–250*	*6* *251–500*	*7* *501–1,000*	*8* *1,001–2,500*	*9* *2,501 & over*
1	12,716	11,937	12,230	12,316	11,246	12,003	15,052	20,437	*
2	5,089	2,293	3,993	2,877	11,688	3,278	3,266	23,769	*
3	9,752	4,724	4,325	4,161	3,821	3,556	3,393	3,229	3,642
4	17,443	7,323	4,304	3,694	3,481	3,199	3,478	4,190	*
5	4,345	2,825	3,135	3,037	3,071	3,132	3,057	3,532	—
6	6,595	5,028	4,615	4,526	3,949	3,725	3,966	*	—
7	8,138	5,841	6,456	7,206	7,905	8,138	8,088	7,197	—
8	5,770	5,625	6,120	6,493	7,154	8,121	8,513	13,848	*
9	17,388	15,781	14,016	13,881	14,421	14,040	13,873	9,452	7,312
10	21,673	35,576	32,191	24,865	25,717	24,860	30,819	30,962	27,783
11	9,601	7,894	6,229	5,934	5,563	5,804	6,539	9,184	9,697
12	8,216	5,136	4,838	4,427	4,266	4,219	3,846	3,378	*
13	6,660	5,545	4,374	4,589	5,159	4,458	4,831	5,993	*
14	6,805	6,188	5,865	6,202	6,665	6,697	6,880	6,809	7,578
15	8,979	8,651	6,759	7,892	12,801	11,824	13,794	12,665	*
16	8,384	7,041	7,115	6,823	6,816	6,901	6,590	6,598	6,614
17	6,554	6,065	6,296	6,584	6,405	6,554	6,163	5,779	5,787
18	6,968	6,074	6,717	5,691	7,160	12,244	14,955	9,454	9,752
19	4,607	4,031	4,522	4,528	4,938	5,322	6,245	5,788	5,867
20	5,877	4,933	4,554	4,477	4,262	5,391	5,432	*	*
All industry groups ...	10,074	7,552	6,772	6,482	6,437	6,421	7,220	8,456	7,918

Source: U.S.A. *Census of Manufactures* 1939, Vol. I, Chapter IV, Table 3.

* Indicates that the figure for this cell has been merged with that for the previous cell to avoid disclosure of information relating to one or two establishments.

— Indicates no observations in the cell.

These results are summarised in Table VI by counting the number of increases between successive size groups within each industry for each classification and arranging the results in the form of a frequency distribution. As was to be expected the *P* classification gives a strong indication of increasing labour productivity in all cases, with 142 increases and only 17 decreases recorded in all the 20 industry groups.[1] The *W* classification gives only 68 increases in all and 79 decreases, and three quarters of the industry groups show 4 or less increases per industry group. Comparing the patterns for each industry group in the light of the classifications adopted in Section II we have the results given in Table VII.

---

[1] The number of decreases between successive size groups for any given industry group will normally be 8 minus the number of increases, since there are only 9 size groups in all. But in some cases, where no figure is shown in the top one or two size groups, the total numbers of differences will be 7 or 6, intead of 8.

TABLE VI

Comparison of the results for value of product per wage earner given by the two bases of classification

No. of increases per industry group	*Frequency (number of groups)*	
	P classification	*W classification*
8	9	—
7	5	—
6	5	2
5	1	3
4	—	7
3	—	2
2	—	2
1	—	3
0	—	1
Total	20	20

TABLE VII

Labour productivity patterns for 20 industry groups based on value of product per wage earner

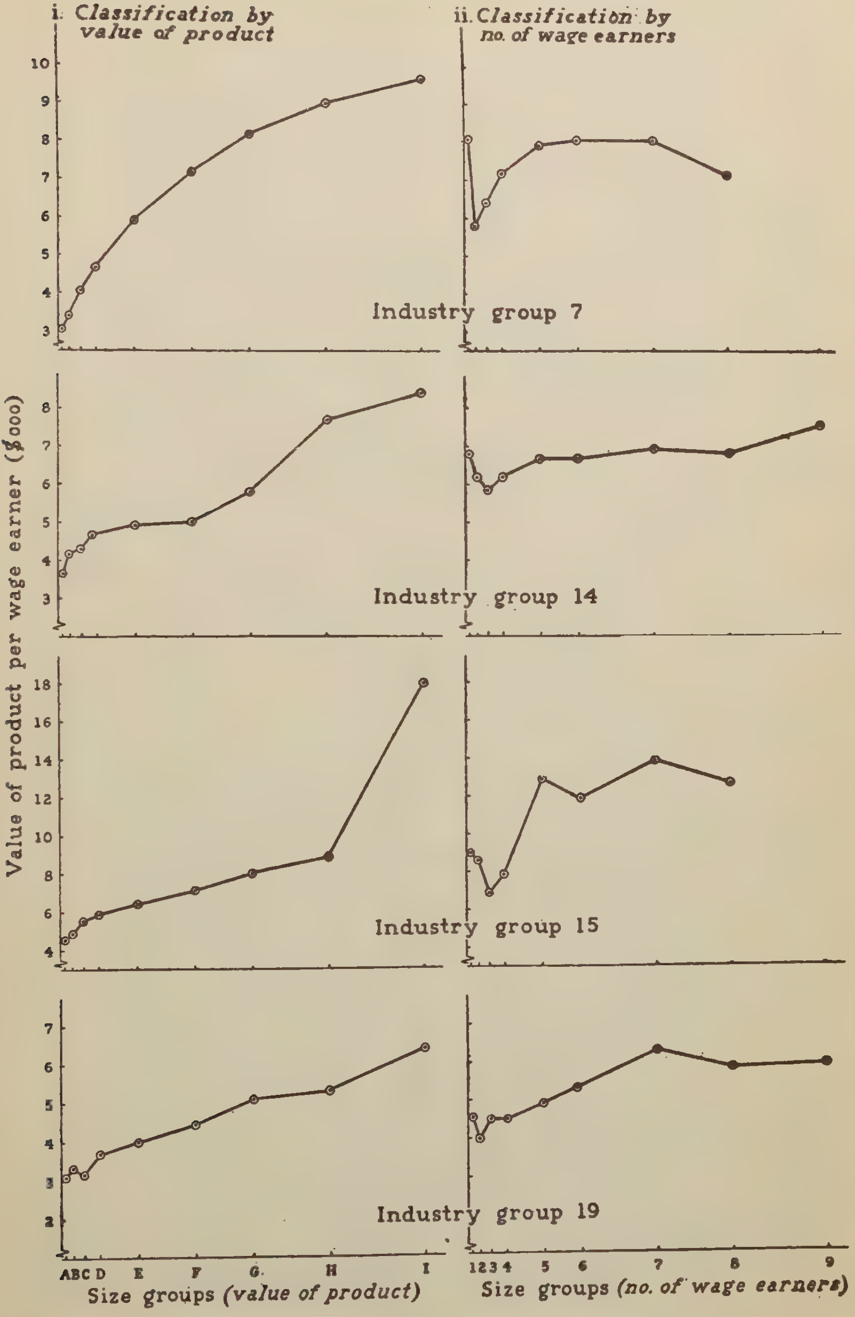
	Pattern	*Industry group*
B (i)	Constant productivity	5, 16, 17
B (ii)	Increasing productivity	1, 7, 8, 11, 13, 14, 15, 18, 19
B (iii)	Decreasing productivity	——
A	No relationship	3, 4, 6, 9, 12, 20

Nine industries fit fairly clearly into the increasing productivity pattern. Industries Nos. 2 and 10, tobacco manufactures and products of petroleum and coal do not fit any of our classifications and further subdivision into more homogeneous groups would be desirable in these cases. Graph I displays some examples of each of the three patterns. Lack of space prevented the correct spacing of the size groups on the horizontal axes, but an attempt has been made to space them in very rough proportion to the actual values, so as not to distort too violently the true shape of each pattern.

It would seem that if only a single basis of classification is possible it is preferable to classify by number of wage earners. This classification alone enables one to distinguish fairly well between cases of constant and increasing productivity: the main difficulty arises where the general tendency of the productivity curve is downwards, as this corresponds to both cases A and B (iii), and only reference to the productivity curve of the *P* classification can satisfactorily distinguish between the two. It seems probable, however, that most of such cases will belong to the category of no relationship rather than that of decreasing productivity.

The theory of Section II applies directly to the interpretation of the patterns of value of product per wage earner. It is of interest, however, to examine briefly the results which appear for two other commonly used indicators of labour productivity, namely value added per wage earner, and wages as a percentage of value added. The detailed figures are given in Tables VIII to XI, and Table XII provides a comparison of the results by showing the number of increases and decreases in productivity between successive size groups for all three indicators on each basis of classification.

Graph 1A. Labour Productivity Patterns



i. *Classification by value of product*

ii. *Classification by no. of wage earners*

Graph 1b. **Labour Productivity Patterns**

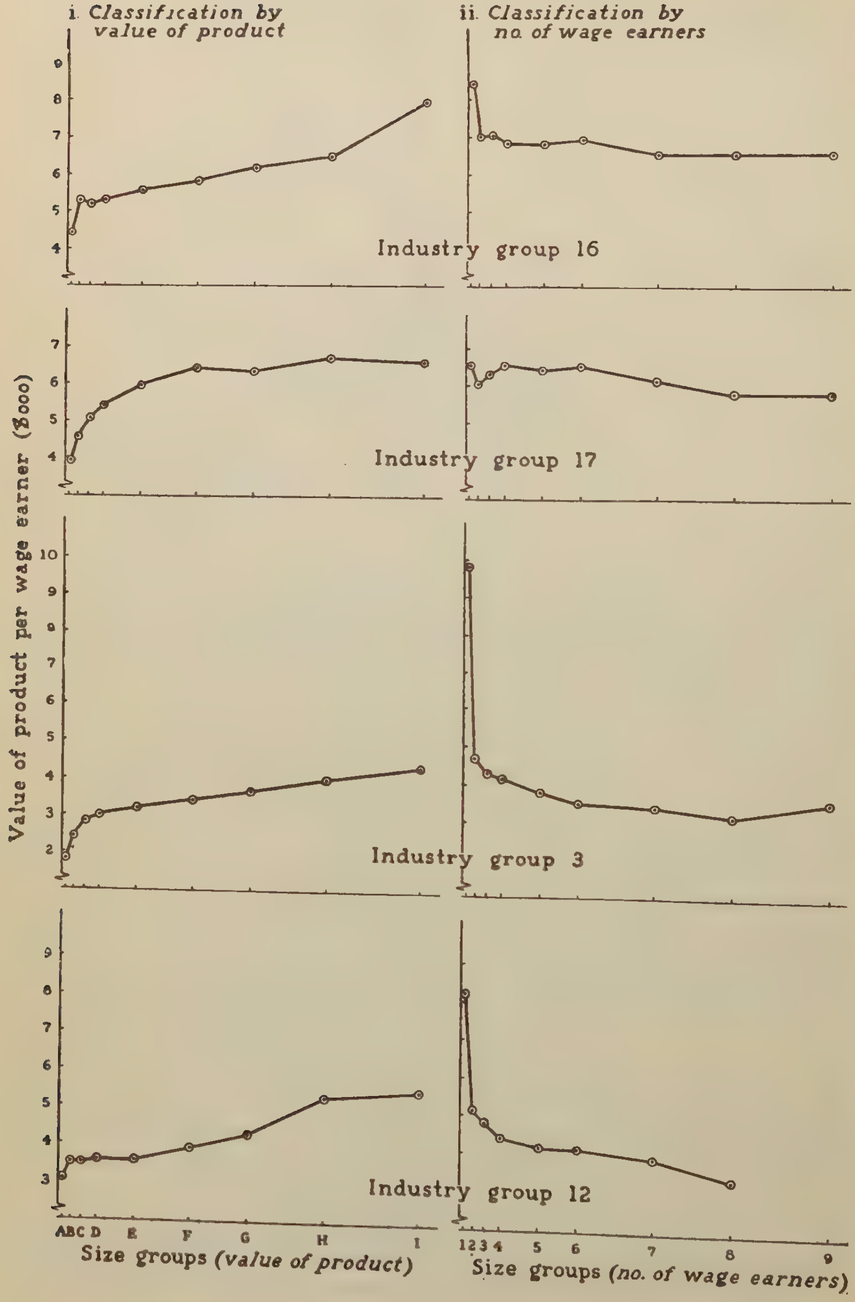


TABLE VIII. *Value added ($) per wage earner, U.S.A. 1939*
Classification by value of product

Industry group	*Size groups (value of product $000)*								
	A *5–*	*B* *20–*	*C* *50–*	*D* *100–*	*E* *250–*	*F* *500–*	*G* *1,000–*	*H* *2,500–*	*I* *5,000 and over*
1	2,590	3,157	3,526	3,567	3,666	4,025	4,751	5,385	5,664
2	1,253	1,110	1,195	1,346	1,601	1,402	1,692	2,441	6,411
3	1,211	1,398	1,444	1,464	1,526	1,533	1,597	1,826	2,159
4	1,278	1,399	1,563	1,770	1,953	2,027	1,965	2,457	2,962
5	957	1,305	1,545	1,599	1,701	1,884	2,134	2,667	*
6	1,793	1,991	2,043	2,000	2,031	2,191	2,389	2,189	3,201
7	1,830	1,886	2,145	2,355	2,656	3,115	3,350	3,807	4,273
8	3,290	3,790	4,119	4,588	5,054	5,114	6,005	6,400	12,377
9	3,567	4,653	5,057	4,970	5,078	5,897	6,475	7,339	7,578
10	2,711	3,086	4,611	4,524	4,424	5,856	5,649	5,817	6,841
11	2,134	2,547	2,774	2,577	2,380	2,576	2,398	2,799	4,293
12	1,682	1,747	1,669	1,632	1,594	1,681	1,806	2,011	2,291
13	2,018	2,276	2,317	2,505	2,535	2,936	3,912	3,742	4,332
14	2,299	2,503	2,484	2,605	2,648	2,738	3,025	3,458	3,247
15	2,854	2,835	2,927	2,922	2,994	3,192	3,451	3,576	4,238
16	2,363	2,810	2,873	2,761	2,825	3,102	3,330	3,845	4,874
17	2,681	3,059	3,221	3,363	3,523	3,837	3,831	4,291	3,812
18	2,193	2,432	2,550	2,674	2,710	3,120	3,081	2,997	3,384
19	1,959	2,035	1,963	2,239	2,287	2,615	2,588	2,936	3,337
20	2,252	2,340	2,408	2,335	2,399	2,656	3,039	5,032	5,082
All industry groups	2,088	2,290	2,454	2,520	2,583	2,713	2,923	3,496	4,180

Source: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Table 7.

TABLE IX. *Value added ($) per wage earner, U.S.A. 1939*
Classification by number of wage earners

Industry group	*Size groups (no. of wage earners)*								
	1 *1–5*	*2* *6–20*	*3* *21–50*	*4* *51–100*	*5* *101–250*	*6* *251–500*	*7* *501–1,000*	*8* *1,001–2,500*	*9* *2,501 and over*
1	4,674	4,244	4,138	4,497	4,111	4,093	4,741	4,135	*
2	2,094	1,242	1,678	1,461	3,762	1,634	1,702	5,616	*
3	3,374	2,142	1,919	1,862	1,723	1,607	1,595	1,505	2,074
4	5,088	2,621	1,910	1,686	1,514	1,434	1,619	2,211	*
5	2,251	1,467	1,601	1,586	1,638	1,855	1,968	2,426	—
6	3,095	2,431	2,176	2,133	2,025	1,907	2,144	*	—
7	3,810	2,830	2,891	3,046	3,397	3,377	3,613	3,125	—
8	4,044	4,015	4,403	4,656	5,021	5,534	5,807	8,963	*
9	8,256	7,188	6,021	5,757	6,571	7,425	8,714	5,330	5,009
10	7,660	8,138	7,691	5,714	5,626	5,380	6,526	7,981	6,524
11	4,722	4,152	3,121	2,761	2,812	2,896	3,038	3,691	4,245
12	3,276	2,154	1,903	1,872	1,760	1,747	1,674	1,743	*
13	3,702	3,063	2,617	2,838	3,377	2,911	3,157	3,993	*
14	3,681	3,204	2,985	2,978	3,123	2,953	2,998	2,938	3,215
15	3,989	3,382	3,200	3,311	3,672	3,547	4,101	3,655	*
16	4,228	3,631	3,524	3,548	3,548	3,985	3,752	4,063	4,140
17	4,091	3,714	3,785	3,896	3,839	3,980	3,828	3,446	3,538
18	3,449	3,036	3,121	2,745	3,292	3,825	4,105	3,180	3,185
19	2,750	2,315	2,502	2,551	2,587	2,737	2,869	3,060	3,365
20	3,539	2,856	2,515	2,527	2,592	3,149	3,688	*	*
All industry groups	4,371	3,362	3,027	2,940	2,902	2,838	3,044	3,340	3,374

Source: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Table 3.

TABLE X *Wages as a percentage of value added, U.S.A. 1939*
Classification by value of product

Industry group	*Size groups (value of product $000)* A 5–	 B 20–	 C 50–	 D 100–	 E 250–	 F 500–	 G 1,000–	 H 2,500–	 I 5,000 and over
1	32.4	29.3	25.9	25.7	26.8	27.3	25.0	23.2	24.7
2	45.4	51.7	51.3	47.5	38.5	43.5	42.3	30.1	14.0
3	54.2	51.0	51.9	52.3	51.4	51.8	52.8	48.2	43.4
4	52.9	59.0	57.3	50.1	44.8	41.5	42.0	45.3	44.5
5	52.3	48.9	46.0	47.1	48.9	52.9	52.8	52.0	*
6	42.1	43.7	42.9	45.0	44.9	42.0	43.5	51.3	43.7
7	38.4	42.6	39.1	38.4	38.5	36.4	36.6	34.0	31.7
8	31.6	32.8	33.6	32.6	30.8	30.9	26.5	27.9	16.2
9	21.5	18.6	18.2	18.7	18.6	18.7	19.2	18.4	19.3
10	30.9	30.5	22.8	23.3	27.9	22.0	25.0	26.2	25.9
11	36.9	33.1	30.9	34.4	38.9	39.5	45.4	42.0	38.1
12	42.7	44.8	48.2	50.6	52.2	50.3	49.9	52.3	46.7
13	40.2	39.9	41.3	41.3	42.7	39.0	32.5	34.6	30.4
14	41.1	41.4	43.0	43.3	43.4	43.8	42.5	39.7	47.4
15	38.6	41.3	40.5	39.4	39.1	37.7	37.1	37.7	33.8
16	38.2	34.6	33.3	36.0	35.8	34.7	35.8	36.1	30.1
17	41.5	40.9	40.1	38.9	36.7	36.1	37.8	34.9	40.0
18	46.5	46.0	45.6	42.3	42.6	38.6	43.0	48.9	49.6
19	49.6	55.5	59.7	57.9	56.5	53.8	55.2	49.3	49.0
20	40.0	40.4	40.8	41.0	41.4	39.2	37.9	29.6	22.7
All industry groups	38.0	39.7	39.0	38.5	38.2	37.9	37.7	34.9	35.4

Source: U.S.A. *Census of Manufactures*, 1939, Vol. I, Chapter IV, Table 7.

TABLE XI *Wages as a percentage of value added, U.S.A. 1939*
Classification by number of wage earners

Industry group	*Size groups (no. of wage earners)* 1 1–5	 2 6–20	 3 21–50	 4 51–100	 5 101–250	 6 251–500	 7 501–1,000	 8 1,001–2,500	 9 2,501 and over
1	21.7	23.2	23.7	24.0	27.2	29.1	27.7	33.5	*
2	34.8	52.1	36.9	42.7	20.0	41.4	41.9	15.3	*
3	27.0	39.4	44.0	44.6	48.2	51.6	52.3	54.9	45.7
4	23.3	39.4	50.8	50.9	50.9	50.2	55.7	53.3	*
5	36.2	45.5	45.5	49.4	53.9	55.0	52.6	54.8	—
6	32.3	39.4	43.1	43.6	44.5	47.6	49.8	*	—
7	24.3	31.8	33.8	35.5	34.8	37.8	34.6	43.2	—
8	28.8	34.3	34.4	33.6	31.9	30.3	29.6	21.7	*
9	11.8	14.1	17.0	18.2	19.0	18.9	17.0	24.9	28.3
10	15.1	15.9	17.6	23.5	27.6	30.7	25.6	23.1	28.4
11	22.0	23.9	31.7	35.9	36.3	38.2	40.9	39.5	43.2
12	28.3	41.1	46.1	47.9	49.5	51.8	55.0	52.5	*
13	28.9	33.6	38.8	37.5	34.5	41.2	40.3	31.5	*
14	30.5	35.5	38.6	40.0	39.9	43.7	45.1	50.5	49.2
15	31.0	35.8	37.5	35.8	33.7	36.6	35.5	38.4	*
16	25.9	29.3	30.0	31.1	32.6	29.5	34.4	35.3	36.1
17	30.8	35.3	35.1	34.5	36.7	36.3	38.9	43.4	43.3
18	32.8	38.2	36.8	43.0	38.6	36.8	38.5	51.4	53.5
19	40.9	52.6	50.7	49.8	53.0	52.8	51.3	51.3	49.5
20	31.4	36.0	39.4	39.4	40.8	35.7	34.6	*	*
All industry groups	24.4	30.8	34.1	35.3	36.5	38.6	38.5	39.1	45.4

Source: U.S.A. *Census of Manufactures, 1939*, Vol. I, Chapter IV, Table 3.

TABLE XII

Summary of results given by three productivity indicators 20 industry groups, U.S.A. 1939

	Indicator					
	Value of product per wage earner		*Value added per wage earner*		*Wages as percentage of value added*[1]	
	+	—	+	—	+	—
P classification	142	17	131	28	89	70
W classification	68	79	72	75	47	100

[1] For this indicator a rise in labour productivity from one size group to the next is indicated by a fall in the value of the indicator, and *vice versa*.

There is little overall divergence between value of product and value added per wage earner: for the second indicator as for the first a marked difference in the proportion of increases to decreases appears on the two classifications, but if the patterns were classified by the criteria of Section II much the same results would emerge as in Table VII. Wages as a percentage of value added, however, shows a substantially smaller number of increases between successive size groups on both methods of classification than either of the other indicators. There is not space to graph the patterns given by the figures in Tables X and XI, but we may say in summary that their outstanding characteristic is irregularity and lack of any clear pattern for either basis of classification taken singly, and we have as yet no theory for the joint interpretation of the patterns, since the bases of classification are neither wages nor value added. If the data were classified by these variables, then the theory of Section II would apply directly to the interpretation of the resultant patterns for wages as a percentage of value added, and it seems probable that smoother patterns would emerge than those given by Tables X and XI.

Our conclusions may be summarised as follows.

(1) The paradox of apparently conflicting results on the relationship of labour productivity to size of establishment is a statistical, rather than an economic matter, the essential condition for the emergence of the paradox being the spread of the original *W*, *P* scatter in relation to the classification grid adopted for the two variables, value of product and number of wage earners per establishment.

(2) Valid inferences about the relationship of labour productivity to size can only be properly established by joint consideration of both patterns in terms of the criteria developed in Section II.

(3) Perhaps the most emphatic warning against trusting either basis singly is provided by Case A, page 344 above, where a zero correlation between value of product and number of wage earners produces steadily increasing productivity on the one classification, and steadily decreasing productivity on the other.

(4) If, however, only a single basis of classification is possible, then it seems preferable to classify by number of wage earners, for the reasons outlined on page 352 above.

(5) If the selected indicator of labour productivity is wages as a percentage of value added, then it is desirable to compute this statistic for establishments grouped by wages and value added, when the theory of Section II can be applied directly to the interpretation of the results.

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APPENDIX

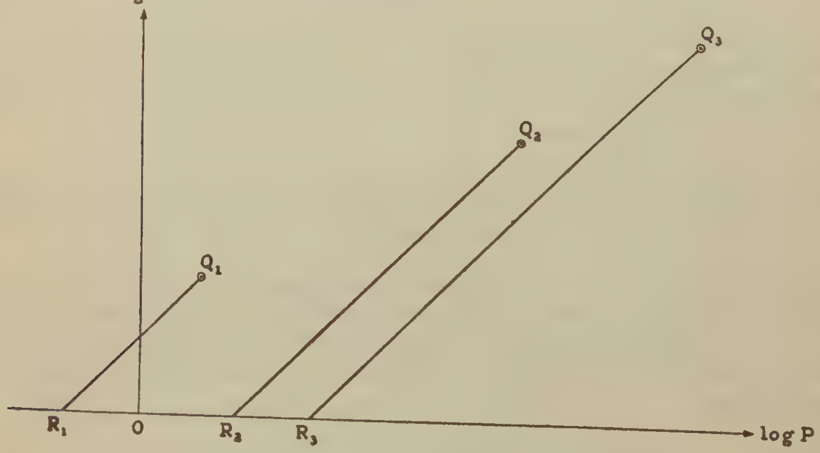
It is desirable to explore more fully the nature of the condition for contradictory results from the two classifications, and in order to compare our results with those of Steindl we work with log P and log W. The log of product per worker is then given by (log P—log W). If we have a series of points with *constant* log product per worker equal to, say, K, then the coordinates of these points satisfy the equation :

$$\log W = \log P - K \tag{7}$$

Thus these points are represented by a straight line with slope unity on the log W, log P scatter, and the constant value of log product per worker is given by the intercept on the log P axis, which is positive if K is positive. If $K = 0$, the units of measurement are such that $P/W = 1$, and a negative value of K corresponds to $P/W < 1$.

If $\log \frac{P}{W}$ is to increase with increases in log P (or log W) we must have a series of points up the log W, log P scatter where the slope of the line joining successive points is always less than unity : the corresponding condition for decreasing labour productivity is a slope greater than unity.

Diagram IX



Drawing parallels at 45° through the three points Q_1, Q_2, Q_3 on Diagram IX gives the successive values of $\log\left(\frac{P}{W}\right)$ as OR_1, OR_2, OR_3.

Assuming approximately linear regressions we see that the condition for product per wage earner to increase steadily on the P classification and decrease steadily on the W classification is that the regression of $\log W$ on $\log P$ should have a slope of less than unity and the regression of $\log P$ on $\log W$ a slope greater than unity (both slopes referred to the $\log P$ axis).

Let

$$x = \log P \text{ and } y = \log W$$

Then the above condition may be written.

$$\frac{E(x-\bar{x})(y-\bar{y})}{E\,(x-\bar{x})^2} < 1 \quad \text{and} \quad \frac{E\,(x-\bar{x})(y-\bar{y})}{E\,(y-\bar{y})^2} < 1 \qquad (8)$$

Define $u = x - y$ = log value of product per wage earner. It was shown on page 341 above that Steindl's condition was equivalent to $b_{xu} < 1$. Now

$$b_{xu} = \frac{E(x-\bar{x})(u-\bar{u})}{E\,(u-\bar{u})^2}$$

$$= \frac{E(x-\bar{x})^2 - E(x-\bar{x})(y-\bar{y})}{\{E(x-\bar{x})^2 - E(x-\bar{x})(y-\bar{y})\} + \{E(y-\bar{y})^2 - E(x-\bar{x})(y-\bar{y})\}}$$

The inequalities in (8) ensure that the denominator of this last expression is greater than its numerator so that $b_{xu} < 1$, and Steindl's condition is verified. The economic implications of the condition, however, are more easily grasped from the following example, which also illustrates how the labour productivity patterns may be derived for a given distribution in the x, y plane.

Let us retain the x, y, u notation and assume that the (x, y) density is uniform over the region R, as shown in Diagram X.

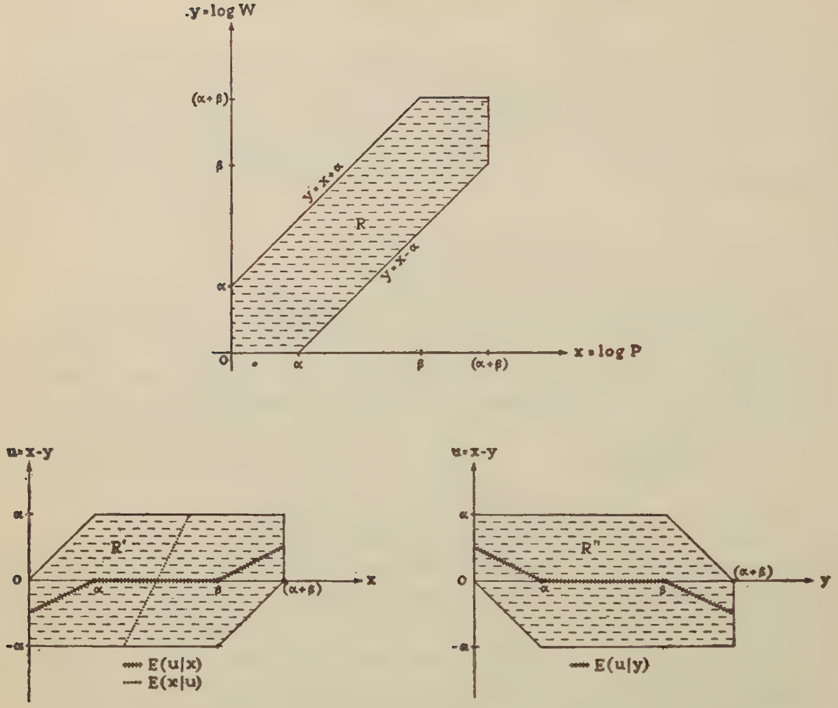
This is an idealised picture of the constant productivity case, in which several simplifying assumptions have been made. The assumption that $f(x, y) = k$ (constant) means that the joint density of W, P is given by

$$f(W, P) = k/PW.$$

This is fairly realistic in that it gives a greater density for the smaller values of P and W. The assumption of an identical range, from zero to $(\alpha+\beta)$, for x and y implies an equal range, from unity to $e^{(\alpha+\beta)}$, for W and P, but the assumption of unequal ranges would only affect the top right-hand end of the diagram. Similarly, the way in which we close the top end of the x, y diagram affects the behaviour of the labour productivity patterns in the higher size ranges, but our main concern is with the conflicting patterns which emerge in the lower size ranges. The final assumption of continuous variables is strictly inadmissible, but it does not distort the results and materially simplifies the exposition.

From the x, y density we can derive two distributions showing log of value of product per wage earner against log of value of product in the one case and against log of number of wage earners in the other. These are shown

Diagram X



in the bottom half of Diagram X. As an example of the derivation of the bounds to the region R' in the u, x diagram we proceed as follows. For any value of x in the range $o < x < \alpha$, $y_{\text{min.}} = o$ and $y_{\text{max.}} = x + \alpha$. Since $u = x - y$, $u_{\text{min.}} = -\alpha$ and $u_{\text{max.}} = x$. Similarly for the range $\alpha < x < \beta$, $u_{\text{min.}} = -\alpha$ and $u_{\text{max.}} = \alpha$, and for the range $\beta < x < (\alpha+\beta)$, $u_{\text{min.}} = x - (\alpha+\beta)$ and $u_{\text{max.}} = \alpha$.

The distribution of establishments over the x, y plane can thus be transformed into a distribution over the u, x plane, and into another distribution over the u, y plane. Under the assumptions made, the u, x and u, y densities will also be uniform over the shaded regions R' and R''. Thus we may write down the regressions $E(u \mid x)$ and $E(u \mid y)$. These are the log labour productivity patterns given by the classification of establishments by log value of product and log number of wage earners respectively.

For $E(u \mid x)$ we have

$$\begin{array}{ll} o < x < \alpha : & E(u \mid x) = \tfrac{1}{2}(x - \alpha) \\ \alpha < x < \beta : & E(u \mid x) = o \\ \beta < x < (\alpha+\beta) : & E(u \mid x) = \tfrac{1}{2}(x - \beta) \end{array}$$

A similar, but reverse, pattern obtains for $E(u \mid y)$ and these two regressions are sketched in the diagram. We may notice the similarity of these patterns to those sketched in Diagram IV.

The u, x diagram also illustrates Steindl's condition. The regression of x on u is

$$E(x \mid u) = \tfrac{1}{2}\{u + (\alpha+\beta)\}$$

and this has slope equal to $\frac{1}{2}$, which satisfies the Steindl criterion, but the diagram shows the horizontal dispersion about this regression to be so great that little significance can be attached to the actual regression equation.

Finally, we may indicate briefly how the labour productivity patterns for the two classifications may be derived in the general case.[1] Let the establishments have some bivariate distribution denoted by $f(W, P)$, where no assumption of continuity is required. From $f(W, P)$ we may derive the two regressions $E(W \mid P)$ and $E(P \mid W)$. The labour productivity pattern given for census data classified by value of product is then

$$\frac{P}{E(W \mid P)} \tag{9}$$

and the labour productivity pattern on the classification by number of wage earners is

$$\frac{E(P \mid W)}{W} \tag{10}$$

It is important to notice that, when we are dealing with P and W in natural units and not in logarithmic form, (9) is *not* equivalent to the regression of P/W on P, for the latter may be written

$$E\left(\frac{P}{W} \,\middle|\, P\right) = P.E\left(\frac{1}{W} \,\middle|\, P\right) \tag{11}$$

and we see that (9) and (11) would only be identical if $E(W \mid P)$ were equal to $1 \Big/ E\left(\frac{1}{W} \,\middle|\, P\right)$; but the former is always greater than the latter, so that the value of product per wage earner given by (9) will always be less than that given by (11), and the shape of the patterns will usually differ also. For the classification by W the two approaches are identical, and (10) is the same as the regression of P/W on W, for the latter may be written

$$E\left(\frac{P}{W} \,\middle|\, W\right) = \frac{1}{W} \,.\, E(P \mid W)$$

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[1] I am indebted to Dr. L. R. Klein for pointing out that, while the argument of this paragraph is correct in terms of generalised regression functions, it is not necessarily true for regressions calculated in the most conventional way—e.g. by a least squares method.

A 'HUMAN NEEDS' DIET: AUTUMN 1954

The long spell of food rationing which began in 1940 affected very differently the cost patterns of different pre-war levels of family diets. Generally speaking, the higher this level had been the weaker was the impact of rising food prices upon the family's actual food expenditure under rationing, partly on account of the restraint imposed upon the quantities purchasable of relatively dear foods, such as meat, butter, eggs, and partly as a result of price control and, with it, the far-reaching elimination of grading. ' Best' qualitites could no longer be bought. Conversely, the more closely the nutritional level before the war had run to the limit of minimum expenditure, the more strongly it was hit by the subsequent rise in average prices, since at the lowest expenditure end the actual prices paid for food had often lain well below the corresponding average prices. The restoration of free markets must be expected to reverse this process at least to some extent. There are already clear indications that this is so ; thus, in the course of 1954, although average food prices had risen slightly, the cost of the ' human needs ' diet, as computed by us, had slightly declined from the spring to the autumn. In the spring of 1954, a ' human needs ' diet for a family of five persons could be obtained at a weekly cost of 63s. 7d.;[1] in the autumn,[2] a similar (though not identical) diet required an outlay of only 62s. 6d.

As usual in this series, our ' human needs ' dietary is computed for a family with three children of school age, and the figure of weekly cost is based on prices collected in Oxford, London, and a number of other towns, mainly in the southern Midlands. The dietary set out in Table I is less detailed than were the former dietaries in this series, when a precise assessment of the food values of the quantities of individual items that rationing and shortages permitted to be bought, was essential. Such detailed tabulation was helped by the fact that although alterations in the controlled—nominally ' maximum '—prices were not infrequent, variations and fluctuations in these prices were very rare, and the price that obtained for a particular food in one place would, as a rule, also obtain during that period elsewhere. Hence it was perfectly reasonable to calculate the precise cost of each single dietary item. Now conditions are changed. There is again a wide choice of foods and a wide variety not only of prices but also of methods of pricing employed in order to attract custom ; consequently it is now far more realistic to assess the cost of typical food groups than that of individual foods, the precise prices of which can no longer be pinned down except with reference to specific instances. Such narrowly circumscribed ' pinning down ' is, however, done in Table II, in order to draw attention to the internal flexibility of the data in Table I; it will not be necessary to repeat a detailed tabulation of this kind in future papers in this series.

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[1] *Cf. A ' Human Needs ' Diet*: *Spring* 1954. BULLETIN, Vol. 16, No. 4.

[2] The second half of October and the first half of November, 1954.

TABLE I

Weekly 'Human Needs' Diet for 5 Persons—Autumn 1954

		Quantity	Average Price per Unit[1]	Changes since Spring	
				Quantity	Outlay
			s. d.		*s. d.*
1	Meat, butcher's	4 lb	2 2	...	...
2	Bacon, boiling	2 lb	2 0	+1 lb 4 oz.	+*1 10½*
3	Sausages or Sausage meat[2] ...	8 oz.	1 9	...	— *0¾*
4	Herrings, fresh or cured	1 lb 8 oz.	1 0	—2 lb 8 oz.	—*2 6½*
5	Eggs	3 eggs	5	— 7 eggs	— *10*
6	Cheese	1 lb 10 oz.	1 10	+1 lb 2 oz.	+*1 10*
7	Milk, ordinary	16 pt.	7	—2 pt.	— *5*
8	*do.*, school	5 pt.	free	...	...
9	*do.*, condensed skimmed	1 tin[3]	10	+ 1 tin	+ *10*
10	Margarine	2 lb 4 oz.	1 4	+ 1 oz.	— *3½*
11	Cooking Fat	—	—	— 10 oz.	—*1 0½*
12	Dripping and Suet	12 oz.	1 0	+ 4 oz.	+ *2*
13	Bread	19 lb 4 oz.	7½	— 12 oz.	— *3¼*
14	Flour, plain	3 lb	1 5	...	— *1*
15	Rolled Oats	2 lb 8 oz.	8½	...	— *1¼*
16	Other Cereals	1 lb 4 oz.	11	— 4 oz.	— *0½*
17	Dried Legumes	2 lb	1 0	...	+ *1½*
18	Potatoes	21 lb	1 1	...	— *4½*
19	Fresh Vegetables	14 lb	4	+ 8 oz.	— *5*
20	Sugar	3 lb 4 oz.	7½	— 4 oz.	— *1¾*
21	Tea	8 oz.	2 10	...	+ *7*
22	Other Foods[4]		3 3½	...	+ *1½*
	Total Outlay		**62 6**		—*1 1*

[1] The price unit is: eggs, one egg; fresh milk, pint; condensed milk, tin; bread, 1¾ lb loaf; flour, 3 lb bag; potatoes, ½ stone; tea, ½ lb; 'other foods', total cost; all other items, lb. [2] Beef. [3] 1⅞ pints equivalent.

[4] ¼ lb cocoa, ¼ lb syrup or jam, ½ lb dried fruit, condiments.

All the data for Table II were obtained at the end of October, within a period of ten days; the figures under each of the headings A and B refer to different districts and—with the exception of the Birminghan data— also to different days. In those cases where the prices of a particular food were in all instances found to be identical with the prices listed for the corresponding group in Table I, all reference to the food is omitted from Table II; as a result, the serial numbers in Table II—covering the corresponding food groups in Table I—show occasional gaps in their sequence.

It will be noticed that we have made some substantial alterations in the structure of the dietary of six months earlier. In particular, eggs and fish are largely replaced by boiling bacon and cheese, and two pints of fresh milk are replaced by a tin of skimmed and sweetened condensed milk, with consequent small adjustments in the amounts of margarine and of sugar, the latter being slightly reduced and the former slightly increased. The additional margarine would be useful in supplying vitamins A and D, lost with the cream of the milk; but even without this adjustment the diet would remain satisfactory. All these are modifications such as a housewife of small means and possessing some experience in cooking and catering would naturally make in order to turn changes in the price structure to her advantage.

TABLE II

Prices of 'Human Needs' Foods in Working-Class Districts
(*October 19—October 29, 1954*)

	Unit	Birmingham		Oxford		London	
		A	B	A	B	A	B
		s. d.	s. d.	s. d.	s. d.	s. d.	s. d.
1*a* Beef, boiling[1]	lb	1 0	1 0	10	1 2	1 2	1 0
1*b* *do*; pieces or minced	lb	2 4	2 4	2 4	2 6	2 6	2 2
1*c* Pork, stewing	lb	1 8	1 10	1 10	1 10	1 8	2 0
1*d* Pig's head[2]	lb	4	6	6	6	6	5
1*e* Mutton, stewing ...	lb	1 8	1 9	1 10	1 9	1 8	2 0
1*f* *do*., roasting	lb	2 8	2 9	3 0	2 9	2 10	2 6
2 Bacon, boiling[3] ...	lb	2 0	1 11	1 10	1 8	1 9	1 9
4*a* Herrings	lb	1 0	11	8	10	10	8
4*b* Bloaters	lb	1 2	1 2	1 2	1 2	1 2	1 0
4*c* Kippers	lb	1 4	1 4	1 2	1 4	1 4	1 2
5*a* Eggs, hens'	egg	4	4½	4	4½	4	4½
5*b* *do*., ducks'	egg	—	4	3½	4½	—	4
6 Cheese	lb	1 8	1 10	1 8	1 8	1 8	1 8
12 Dripping	lb	10	9	1 0	1 0	1 0	10
14 Flour, plain	3lb bag	1 5	1 4½	1 4	1 4½	1 4	1 5½
15 Rolled Oats	lb	8½	8½	8½	7½	8	8½
16*a* Semolina	lb	11	10	9	9	9	10
16*b* Macaroni...	lb	1 0	1 2	1 0	1 0	1 0	1 1
16*c* Rice	lb	1 0	1 0	1 3	1 0	10½	1 3
16*d* Barley	lb	7½	8	7½	8½	8	7½
17*a* Peas	lb	1 0	11	1 0	1 0	10	10
17*b* Beans	lb	1 0	1 0	1 3	10	11	1 0
17*c* Lentils	lb	11	10	1 0	10	11	11
18 Potatoes	½ stone	1 0½	1 0½	1 0½	1 1	1 0½	1 0½
19*a* Onions	lb	5	5	4	5	5	4
19*b* Greens	lb	4½	4½	3	4	4	3
19*c* Swedes and Turnips ...	lb	3	3½	3	3½	2½	3
19*d* Carrots	lb	4	4	5	4½	4	4
20 Sugar	lb	7½	7½	7½	7	7½	7½
21 Tea	½ lb	2 10	2 9	2 10	2 8	2 8	2 10
22*a* Jam, plain	2 lb	1 9	1 10	1 10	1 10	2 0	1 10
22*b* Marmalade	2 lb	1 10	1 11	1 10	1 11	2 0	2 0
22*c* Raisins	lb	10	11	8	11	10	10
22*d* Currants	lb	11	11	11	11	11	11
22*e* Dates, stoned	lb	—	10	10	10	10	—
22*f* Apples, cooking ...	lb	4	4½	3	5	4	3

[1] The serial numbers refer to the groups in Table I.

[2] Selling in ½ heads of 5 to 6 lb.

[3] Fat and lean. Pieces of fat bacon only could be bought at prices lower than shown in the Table (from 8d. to 1s. a lb).

Boiling bacon at about 2s. a lb.—generally fairly fat corner pieces—is not a particularly cheap food from the nutritionist's point of view. However, bacon is valued by housewives because of its many culinary uses, and as the prices of all bacon had declined slightly since spring and low-priced boiling bacon become more plentiful, there existed a strong inducement to buy more of it in substitution for other foods. The fat of bacon partly replaces cooking fat. As compared with suet and dripping—both now freely available—cooking fat had become dear and we omitted it from our autumn dietary in favour of these two other fats. The two items in the dietary, the prices of which increased substantially in the course of the six months, were eggs and tea. It is possible that slightly cheaper brands of tea, at 2s. 8d. for

8oz., could be obtained generally if one carefully searched for them; yet in many inexpensive shops the lowest price appeared to be 2s. 10d.

It must be stressed that—in contrast to the Tables of minimum prices presented in some previous articles in this series[1]—the prices in the columns of Table II were not scattered over the period of the survey and over many towns, but obtained, in each instance, in one shopping district and on one arbitrarily selected day. They were quoted in shops known to be inexpensive and at market stalls; but apart from the fact that notice was taken of foods exhibited openly at prices that evidently were advantageous, no attempt was made to trace for any particular food the minimum price which assiduous search and repeated comparison of prices might have disclosed. Hence the prices as set out can be considered normal purchase prices at which any economical shopper would have been able to obtain these foods; at the same time they indicate the variations and fluctuations that exist from shop to shop, and that may occur from week to week even in the same shop. But when one item becomes a little dearer another will generally be offered a little cheaper, without, in the short run, more than at most a very transitory effect on the purchase cost of the diet.

The dietary in Table I could provide a family of the assumed structure with satisfying and nutritionally satisfactory meals.[2] The replacement of a tin of skimmed condensed for two pints of fresh milk reduces the total amount of milk by only ⅞th of a pint, while the additional 18oz. of cheese represent the milk equivalent of between 6 and 7 pints. The substitution of cheaper for dearer margarine—1s. 4d. a lb. as against 1s. 6d.—seemed justifiable in view of the satisfactory quality of several samples of the lower priced margarine, which appeared to be as good in every respect as the general type of top-price margarine under rationing.

No strict comparison is possible between the 1936 'human needs' dietary suggested by the late Mr. B. S. Rowntree and this post-rationing dietary, as the changes in dietary habits in the course of the last fourteen years have been too drastic. Mr. Rowntree's dietary allowed for the consumption of considerably more meat foods but of considerably less milk, cheese, and eggs.[3] The total weekly cost of his dietary, if bread was purchased and not baked at home, amounted to just under 22s. No further figures of the cost of 'human needs' nutrition are available after that until our first survey, undertaken in the spring of 1941.[4] From then onward, such figures have been published twice annually in the BULLETIN.[5] It is interesting to consider these figures of cost in relation to the index figures of the retail prices of food. The index of retailed prices given monthly in the Ministry of Labour Gazette is not continuous over the whole period, owing to changes in its internal structure and in weighting, in 1947 and 1952; but the London and Cambridge Economic Bulletin publishes an unbroken series of annual indices that reaches back th 1919, the figures from 1938 onward—which to us are of special interest—being based on the relevant Ministry of Labour publications and on the National Income White Papers. These index figures represent monthly averages per calendar year. We have used them as they stand for plotting a Diagram, setting them against indices obtained from the averages of the cost figures of the 'human needs' diets for the spring and the autumn of each year.[1]

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[1] BULLETIN, Vol. 15, No. 12, p. 425 and Vol. 16, No. 4, p. 133.

[2] The intake of essential nutrients would be virtually the same as that from previous diets suggested in this series. *Cf.* BULLETIN, Vol. 16, No. 4, *op. cit.*, *et ante*.

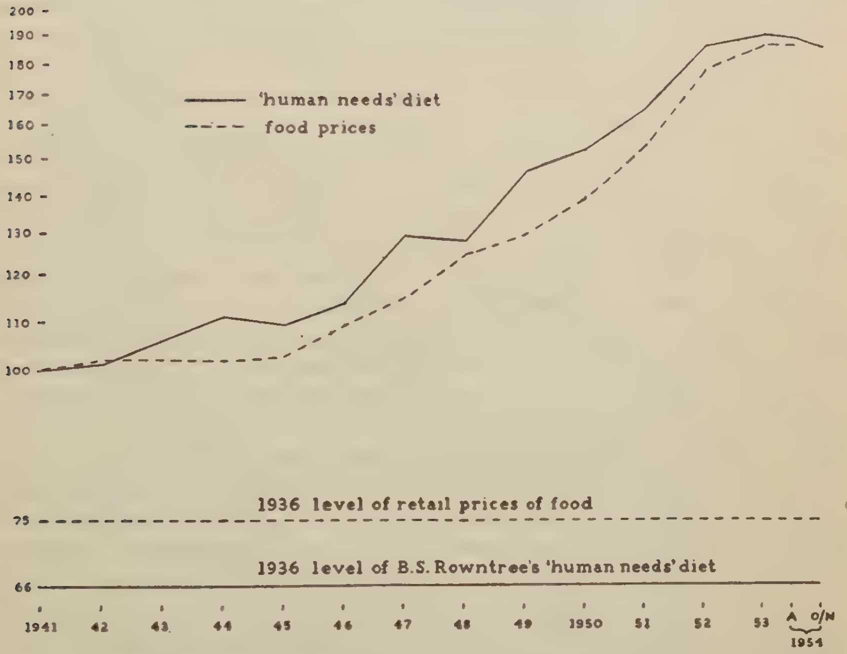
[3] *Viz.*, 9 lb. 8 oz. of meat, bacon, and sausages, 2 lb. 8 oz. of herrings and kippers, 14 pint equivalent of skimmed condensed milk, 1 lb. of cheese, 1 egg.

[4] T. Schulz, *A 'Human Needs' Diet in War-Time.* BULLETIN, Vol. 3, No. 5.

[5] Except in 1947, when a survey was undertaken only in the spring.

The data are plotted on a semi-logarithmic scale, taking 1941 as the base year for both series of indices. The resulting picture is very instructive: with

Indices of cost of 'human needs' diet & of retail prices of food



the firm establishment of rationing, the 'human needs' cost of nutrition became firmly established above the relative level of food prices, even though in subsequent years changes in the supply of specific foods did on occasion inverse the trend of cost in opposition to the rising trend of actual food prices.[2]

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[1] For 1954 the graph is based on the specific figures for April and October-November, for the human 'needs' cost index, and on the figure for the month of April, for the index of food prices, since for that year an annual average of the index of food prices was not yet available.

[2] The 'human needs' cost index for 1942 is held down mainly as a result of the exceptionally large ration—8 oz.—per head of inexpensive cheese in the autumn of 1942 and the consequent adjustments in the quantities of more expensive animal protein foods. This reduced the total cost of the diet from 34s. in April 1942 to 32s. 2½d. in November.

From 1949 onward, however, a distinct retardation in the rise of the ' human needs' cost relative to the rise in food prices becomes apparent, reflecting the gradual revival of free markets, with the return to competition and the grading of qualities; it will be noticed that the angle of the ' human needs ' cost curve between two points has now become consistently less acute than the corresponding angle of the curve of food prices.[1]

It will however be observed that in relation to their respective 1936 indices the ' human needs ' index still appears well above the index of food retail prices.[2] But from a nutritionist's view point the 1936 ' human needs ' dietary is inferior to the dietaries since 1941, since it allows for the consumption of much less and then only skimmed milk.[3] If in the autumn dietary for 1954, we replaced all the milk purchased as such by the equivalent amount of skimmed condensed milk (making adjustments for differences in the nutritional values of these two foods, in the quantities included in the dietary of sugar, margarine, flour, and cereals), the total cost of the dietary would be reduced from 62s. 6d. to about 59s. 3d. This would bring the ' human needs ' index within 6 per cent of the index of food prices, on the 1936 base. This calculation may help to set the two series of indices in a clearer perspective; yet it is of little practical value, because it half-way transposes the index of ' human needs ' cost into a price index, and for a country with a fairly high standard of living, no ' human needs ' price index can be established. For, taken by themselves, very few foods in a realistic ' human needs ' dietary are indispensable or, at least, are essential in rigidly fixed quantities. The ' average housewife '[4] catering above the ' human needs ' standard, also will make considerable adjustments in her dietaries over a period—in the short run, in order to offset unfavourable changes in prices, and in the longer run, through changes in nutritional habits. But the wider the range of foods included in a dietary and the larger the total expenditure on it, the smaller on the whole will be the impact of such adjustments on the index of cost in relation to the corresponding index of prices.[5] The ' human needs ' dietary, on the other hand, with its relatively narrow range of foods, can be highly sensitive to alterations in its composition, in its figures of total cost independently of changes in food prices. Yet by its ' human needs ' character it is firmly wedded to such dietetic or nutritional adjustments : a ' human needs ' dietary must provide a selection of inexpensive foods that people will normally eat and it must provide for a nutritional intake that at the time to which the specific dietary applies, is considered satisfactory; but subject to these qualifications, the composition of the dietary is highly elastic. The changing pattern of our dietaries assumes indeed that a housewife is capable of adjusting her menus so as to profit by changes in supply and prices as circumstances permit; yet even so the actual degree of individual adaptability of skill and taste required is less than our schedules may imply, since, for any given period, our schedule represents only one out of a considerable number of possible ' human needs ' dietaries, all of them realizable within the computed limits of total cost. The detailed data in Table II will have helped to make this clear. To replace with reference to a ' human needs ' diet the cost index by a price index would imply the nonsensical assumption that the particular dietary from which the index derives, represents in every detail an unalterable standard of ' human needs '.

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[1] The index figure of ' human needs ' cost for 1953 used for the graph is derived from the figures of actual cost in the spring and the autumn of that year, omitting, however, the ' special ' allowance of about 3 per cent of the total cost in the autumn, which then was included so as to allow for errors in selecting the right foods by housewives unaccustomed to free markets.

[2] Marked in the Diagram by the two horizontal lines cutting the index scale at 66 and 75.

[3] Cf. *supra*, p. 366.

[4] *I.e.*, the ' housewife ' assumed to be representative of the average family of her group.

[5] Under the impact of rationing the ' weights ' employed for the computation of the accepted indices of food prices—the Ministry of Labour index as well as the index of the London and Cambridge Economic Bulletin—did, of course, become highly unrealistic; but if subsidies on ' overweighted ' foods helped to keep the price index down, rationing of the more highly prized foods and the enforced use of inferior and relatively cheaper substitutes —margarine for butter, dried skimmed milk for fresh full cream milk, dried eggs for shell eggs, etc.—tended to exercise an influence in the same direction on the actual cost of food consumption.

T. SCHULZ.

APPENDIX

‘HUMAN NEEDS’ DIETS IN NORTHERN IRELAND

Below we are showing two ‘human needs’ dietaries which are adapted to suit the consumption habits of working class families in Northern Ireland; they thus include substantially larger quantities of potatoes and vegetables, and smaller quantities of the more expensive cereal foods and of fresh vegetables, than our English ‘human needs’ dietaries. Mainly owing to this and also to the lower price of fresh vegetables in Belfast as compared with the price in the English Midlands, the cost of the Irish diets is somewhat less than the cost of the corresponding English diets. The cost of the Irish spring dietary was computed at about the same time as that of the English spring dietary; that of the Irish autumn dietary, however, is derived from prices that ruled early in October, while those of the English dietary in the preceding article applied nearly a month later, when in particular eggs had become generally dearer. Thus although a direct comparison of the respective cost figures shows the cost of a ‘human needs’ diet in Belfast in the spring of 1954 *as about* 2 *per cent below the cost of a similar diet in the English Midlands, but in the autumn by as much as* 5 *per cent below it, this latter figure almost certainly over-estimates the difference in the actual cost of the two diets.*

T. S.

This is an attempt to estimate the cost of a ‘human needs’ diet in Belfast in spring and autumn of 1954. The methods and standards are modelled on those of Miss Schulz, but the dietaries have been compiled in keeping with local patterns of consumption.

In Northern Ireland most school-children get two-thirds of a pint of free milk daily as compared with one-third of a pint in England. In a family with 3 school-children the weekly expenditure on milk is thus lower in Belfast by the price of five pints of milk.

The prices were collected during the first two weeks of April and of October. In the intervening months meat prices rose sharply after decontrol and then fell to their present level. The cheaper cuts are now slightly below their previously controlled price, and there is a better range of minced and sausage meats. This, together with the availability of local herrings during October, makes the autumn expenditure on meat and fish lower than in the spring. Cheaper margarine and dripping and the seasonal decrease in vegetable prices also make the autumn cost lower, and counteract the rise in the price of bacon, eggs, and tea.

D. E. Field.

TABLE III. *'Human Needs' Diets for 5 Persons in Belfast*

	Spring 1954		Autumn 1954	
	Quantity	Average Price per Unit[1]	Quantity	Average Price per Unit[1]
		s. d.		s. d.
(*a*) *Meat and Fish*:				
Butcher's Meat	6 lb	2 3	4 lb	2 2
Tripe	—	—	1 lb	1 0
Meat Roll	—	—	1 lb	1 8
Bacon	1 lb	2 6	8 oz.	3 6
Kippers	1 lb	1 6	—	—
Herrings	—	—	2 lb 8 oz.	10
Outlay on Group (*a*)		*17 6*		*15 2*
(*b*) *Dairy Produce*:				
Eggs	10 eggs	3 0	9 eggs	4 0
Cheese	8 oz.	2 4	8 oz.	2 2
Milk, ordinary	13 pt.	6½	13 pt.	7
do., school	10 pt.	free	10 pt.	free
Outlay on Group (*b*)		*10 8½*		*11 8*
(*c*) *Fats*:				
Margarine	2 lb 8 oz.	1 6	2 lb 8 oz.	1 4
Dripping	10 oz.	1 6	5 oz.	1 0
Cooking Fat	—	—	5 oz.	1 6
Outlay on Group (*c*)		*4 8¼*		*4 1½*
(*d*) *Cereal Foods*:				
Bread	23 lb	7½	23 lb	7½
Flour	3 lb	6½	3 lb	6
Rolled Oats	2 lb 8 oz.	8½	2 lb 8 oz.	8½
Barley	8 oz.	6	8 oz.	7
Custard Powder	8 oz.	2 0	—	—
Rice	—	—	8 oz.	1 3
Outlay on Group (*d*)		*12 4¾*		*12 5¼*
(*e*) *Pulses*:				
Dried Peas	12 oz.	1 0	12 oz.	1 6
Lentils	12 oz.	1 0	12 oz.	10
Outlay on Group (*e*)		*1 6*		*1 9*
(*f*) *Vegetables*:				
Potatoes	28 lb	2 0	28 lb	2 0
Fresh Vegetables	10 lb	4[2]	10 lb	3[3]
Outlay on Group (*f*)		*7 4*		*6 6*
(*g*) *Other Foods*:				
Jam	1 lb	1 0	1 lb	1 0
Sugar	3 lb 8 oz.	7¾	3 lb 8 oz.	7½
Cooking Apples	—	—	1 lb	4
Oranges...	1 lb	1 0	—	—
Tea	8 oz.	4 8	8 oz.	5 8
Condiments	—	1 6	—	1 6
Outlay on Group (*g*)		*8 1¼*		*7 10¼*
Total Outlay		**62 2¾**		**59 6**

[1] The price per unit is: eggs, dozen; milk, pint; bread, 1¾ lb loaf; potatoes, stone; condiments, total outlay; all other foods, lb. [2] Per lb.: cabbage, 3½d.; carrots, 4d.; onions, 5d. [3] Per lb.: cabbage, 2d; carrots, 6d.; onions, 6d.; turnips, 1d.

RETAIL TRADE, RETAIL PRICES AND REAL WAGES IN U.S.S.R.[1]

1. It is commonly claimed that the Soviet economy is good at investment and armament but bad at consumption. Bad at agriculture it most certainly is, but the official figures show an immensely rapid growth after the war in consumption nevertheless. This article examines these claims, and finds them only slightly exaggerated. In summary, the consumption of 'workers and employees', which is roughly co-extensive with the free population of towns, workers' settlements and state farms, increased as shown below. For comparison series are also given for some of the other main components of the Soviet national income. In each case the connexion between the pre- and post-war data is the least certain part, and except for the population it is not clear how the new territory acquired in 1940 has been treated in that year.

The reader is most particularly advised to refer to the sources and the critiques of the sources before using any of these figures. The most surprising thing is undoubtedly the fluctuations in real wages, and their failure to surpass the pre-plan level after so much industrial expansion. Part of the answer can be found simply by paying more attention to detail: the volume of consumer goods produced per head has not risen anything like so fast, even according to official claims. And these in turn are exaggerated by the use of the celebrated 1926–27 prices (see footnote 1, page 388), though not so much as in the case of the total index of production. Again the official series omits the very numerous products of small craftsmen, a great feature of old Russia, which practically vanished between 1928 and 1932. Moreover we must take into account that imports of consumption goods fell by about 40 per cent between 1928 and 1938,[2] and a good deal further again by 1940. Then much food production is in no way processed for urban consumers and does not pass into the index of industrial production; to which extent weight must be given to the index of agricultural production, which has not kept pace with the population. Also food exports did not fall quite so far as consumer good imports,[2] but this is offset by the loss of horses, which eat grain that goes to swell the gross index of agricultural production.

It must be admitted that even all these qualifications fail to reconcile completely the series for production and consumption. In the last resort, then, we must look to a further error on one side or the other. The real wage figures are unique in that one element in their calculation, the retail price index, rests in part on foreigners' observations. This makes them, in the writer's opinion, more reliable. We return to this problem for 1940 and the post-war years, when the discrepancy is more tractable, in section 6.5, but it is beyond the scope of this article to effect a reconciliation for the huge gap in 1928–40.

[1] The author wishes to acknowledge the assistance of Dr. Jasny and Messrs. Nove and Seton; and of Mr. Winsten in the matter of price index formulae.

[2] A. Baykov, *Soviet Foreign Trade*, pp. 102–5.

TABLE I

Summary Data

	Population (millions, mid-year)	Industrial Production (gross, official)[1]			Agricultural Production[3] (gross, official)	Real volume of investment (official)[1]	Real wages[4]	Volume of State and Co-op. Trade[1]	Retail Prices[4]
		Total	Consumer goods only	Total (net, Hodgman)[2]					
1928 ...	151[5]	14	18	25	89[8]	—	202	44[12]	8.8
1937 ...	165[5]	69	75	84	92[9]	—	124	—	62
1940 ...	172–196[5,6]	100	100	100	100	100[11]	100	100	100
1947 ...	193[7]	93	—	87	—	88	c. 46	c. 67	—
1948 ...	195[7]	118	99	108	—	109	93	75	193
1949 ...	198[7]	141	107	130	106[9]	130	107	90	166
1950 ...	201[7]	173	123	150	114[9]	161	131	117	142
1951 ...	205[7]	202	143	172	—	180	144	134	136
1952 ...	208[7]	227	158	—	110[10]	200	160	149	126
1953 ...	211[7]	251	176	—	—	208	172	179	—

[1] As in *Gosplan* yearly results; Malenkov, *Pravda*, October 6th, 1952, etc.

[2] In *Soviet Economic Growth*, ed. Bergson, pp. 232, 236. Physical outputs of rather few commodities weighted by their wage bills in 1934. The official indices of agricultural and industrial production are the (very badly) deflated sum of the gross output values of all factories or farms.

[3] 'Sel'skoe Khozyaistvo.'

[4] As calculated below.

[5] F. Lorimer, *Population of the Soviet Union*, pp. 135, 188.

[6] New territory (which at the end of 1940 was roughly the same as, though slightly less than, in 1947–53).

[7] Writer's estimate; closely corresponds to other unofficial estimates.

[8] N. Jasny's unofficial computation: *The Socialized Agriculture of the U.S.S.R.*, p. 682.

[9] Shabalin, *Voprosy Ekonomiki*, 8/1953, p. 55. It should be noted that any index showing the excellent year 1937 as less than 1940 is highly suspicious.

[10] Khrushchov, *Pravda*, September 15th, 1953.

[11] The relation of this year to the post-war series is much higher here than in the official estimate. Our figure comes from Nove, *Review of Economics and Statistics*, February, 1954.

[12] *All* trade as a percentage of all trade in 1940 (Orlov, *Sovetskaya Torgovlya*, 2/1954). It would not be possible to separate out state and co-op. trade in this period owing to organisational changes.

The astonishing rise in real wages, 1947–8, is due to the famine that followed the failure of the 1946–47 crops. The rise after 1948 seems to be mainly due to the increased supply of durables. Food lags a little and textiles lag a great deal (compare Table VIII). It may seem surprising that food does not lag more, but it appears that the procurement system—which feeds the urban population, which alone we are studying—is working better than before the war.[1] The main agricultural deficiency is in industrial crops, fodder crops and hides, and possibly in food consumed on the farm—which does not concern this article. Imports have little influence (section 6.5).

Some may take heart from the Soviet failure to restore the citizen's well-being to the pre-plan level (1928). Others however will be disconcerted at the extremely rapid rise of real wages since the war, a rise continuing after mere reconstruction was over; also at the official figures' consistency (since 1940) and apparent freedom from gross exaggeration. Common opinion notwithstanding, the new rulers will be hard put to it to raise real wages as swiftly as Stalin did in some years.

For the rest our procedure is as follows. In section 2 the official sources are criticised : (2.1) the volume index of retail trade, (2.2) the price cuts, (2.3) the money value of retail trade, (2.4) the official price index. In section 3 we introduce two ' bourgeois ' concepts, (3.1) factor cost and (3.2) the rate of turnover tax; in 3.3 we draw certain conclusions about the course of these magnitudes, and in 3.4 we define retail trade more closely. Section 4 establishes the course of money wages. Section 5 returns to the cost of living; 5.1 criticises Mrs. Chapman's index and 5.2 sketches in the year 1940. Section 6.1 sets out the course of real wages, and 6.2 certain qualifications. 6.3 discusses the Soviet claims as to real wages, 6.4 and 6.5 present certain checks.

2.1 As usual we must exhaustively criticise our sources. The *volume index of state and co-operative trade* appears in the *Gosplan* results yearly (Moscow papers, end of January) and from time to time in the speeches of bigwigs. Chained, it produces the results of column 3, Table II. Since it is not so sensitive a datum as the real wage index it is presumably less subject to falsification. Nevertheless the 1948–1947 link in the chain is missing, and must be rather hazardously deduced (Table II, note 20); the period 1951–4 shows some slight exaggeration (Table II, note 5); and there is grave doubt as to the connexion with 1940. The *Gosplan* results for 1948 claim that volume was ' higher ' than in 1940, and no such claim was made in 1947. Put then 105 per cent of 1940 in 1948. But Mr. Mikoyan[2] says the volume index in 1953 was 179 per cent of 1940, and chaining back to 1948 this gives 75 per cent of 1940 for that year. Mr. Mikoyan is generally thought to be reasonable and moderate—he has recently said that some capitalist trading methods are superior and should be imitated; he was speaking at a later date; he was speaking after Stalin's death, when he could more easily get away with revised figures; and his figure is lower. Moreover he is supported by the most distinguished of Soviet writers on trade, M. M. Lifits,[3] who after

[1] Cf. United Nations, *Economic Survey of Europe in* 1953, pp. 268–9.

[2] *Pravda* 27 April, 1954. [3] *Voprosy Ekonomiki*, 7/1951.

TABLE II

Data on State and Co-operative Trade in Goods

	Planned value of 'economy' on yearly basis (R.md.)[1] (1)	Date of cut (2)	Official volume index[2] (3)	Current turnover money value of (R.md.) (4)	Price indices (IV/47 = 100)		Turn-over tax levied[4] (R.md) (7)	(7) as per cent of (4) (8)	—very approximate—		
					deduced[3] (5)	official (6)			Factor cost of production of goods[5] (R.md.) (9)	Unit factor cost as index[6] (10)	Rate of turn-over tax[7] (%) (11)
1940	—none—		133[8]	175[13]	38.4	36[17]	106	58.8	108	51	62
1944		,,		143[13]			95	66.6			
1945		,,	?59	(174)[14]	86		123	(70.6)			
1946		,,	?77[9]	(256)[14]	97		191	(74.6)			
1947 (actual)	none, but monetary reform on Dec. 14		?90[10]	(308)[14]	100	100	240	(78.6)	138	93	122
1947 (planned)			—	325[15]			255	78.6			
1948	67	Dec. 14, '47	100	(332)[14]	96.6		247	(74.4)	159	100	109
1949	59.5	March 1	120	(348)[14]	81.5		246	(70.8)	176	92	98
1950	95	,,	156	(356)[14]	66.4	II/1950=57.8[18]	236	(66.2)	191	77	87
1951	27.5	,,	178[11]	(385)[14]	62.9		248	(64.5)	211	75	83
1952	24	April 1	198[11]	(391)[14]	57.3		247	(63.0)	218	69	80
1953 (actual) ...	50	,,	238[11]	435[16]	54.4	51.3[19]	244	56.1	264	70	65
1953 (planned)	46		—	405.5[16]			241	59.5			
1954 (planned)	20	,,	273[12]			44.8[20]					

[1] Source : *Pravda* leader on given date. In 1948, 1949 and 1950 adjusted to include co-op. trade as explained in text.

[2] Source : *Gosplan* yearly results (*Pravda*, end. Jan. or beginning Feb.) ; increase of each year upon the one before, chained.

[3] Column (4) divided by column (3), on a base explained in the text.

[4] Source : Budget discussion in *Pravda* or '*Hansard*'.

[5] Column (4) minus 63 per cent of column (7) in 1940 ; thereafter 70 per cent. See Appendix I.

[6] Column (9) expressed as an index, divided by column (3).

[7] 63 per cent of column (7) as per cent of column (9) in 1940 ; thereafter 70 per cent. See Appendix I.

[8] 1953 = 179 on 1940 = 100 : Mikoyan , *Pravda*, April 27th, 1954.

[9] The 1953 volume was more than three times that of 1946 (Orlov, *Sovetskaya Torgovlya* 2/1954). See also note 10.

[10] Guessed : the increase 1948–7 was ' considerable '. This is the cause for the doubt as to the 1946 and 1945 figures. See also note 9.

[11] According to Mikoyan, *loc. cit.*, the 1954 plan figure is 175 per cent of 1950. But the annual results chained give 180 per cent. I have therefore adjusted these three intermediate figures downwards.

[12] Mikoyan, *loc. cit.* [13] Prokopovicz, *Histoire Economique de l'U.R.S.S.*, p. 466.

[14] Interpolated from column (8). [15] Given straight in 1947 plan : *Pravda* March 1st, 1947.

[16] Explained in text (sec. 2.3).

[17] Price indices, II/1954, on 1940 = 100, are : groceries (apparently in state and co-op. trade only) = 114, manufactured goods = 127 (Mikoyan, *Pravda* April 27th, 1954). Weights from Mikoyan, *Pravda* October 25th, 1953 : manufactured goods were 36.9 per cent of *all* trade in 1940, and are 45.3 per cent of it to-day. Take Kolkhoz trade to be 16 per cent of all trade, and to involve no manufactured goods. This gives, for state and co-op. trade, weights of 43/57 in 1940 and 53/47 in 1953. Formula : Fisher's Ideal. There results an index of 120 for II/1954, which can then be chained onto the 43.3 of note 20.

[18] Prices in 1954 are 75 per cent of those in 1950, i.e., of the 43.3 in note 20 : the context makes it clear that the second to fourth quarters are meant in each case (Zverev, *Pravda*, April 22nd, 1954).

[19] The volume of *all* trade in I/1954 was 30 per cent greater than in I/1953, and in money terms 15.5 per cent greater. (Mikoyan, *Pravda*, April 27th, 1954). Assume, reasonably, that Kolkhoz market prices followed the others exactly. Now I/1954 is much the same as II/1953, for which we have data in note 20. Then if II/1953 = 50, II/1953 = 56.4. Weighted average (79 and 21) of these two figures gives 51.3.

[20] Weighted average (21 and 79 respectively) of price indices for II/1953 and II/1954. These are 50 (Malenkov, *Pravda*, Oct. 6th, 1952) and 43.3 (Mikoyan, *Pravda*, April 27th, 1954).

giving us a series for the post-war trade volume index (omitting as usual the 1948–47 link) says ' In 1950 the volume of state and co-operative retail trade considerably exceeded that of 1940 '. In its context this means that it did not do so in 1949. Lastly the indices of physical production support the lower estimate (section 6.5). For all these reasons I have accepted Mr. Mikoyan's relation between pre- and post-war trade volume.

Should a still more moderate claim be made in the future much of this article would have to be revised. That this series has caused the authorities some embarrassment is shown by Mr. Malenkov's failure to mention it at all in his very full account of economic progress since 1940 to the last Party Congress.[1]

The Soviet formula for this index is $\frac{\Sigma p_0 \, q_1}{\Sigma p_0 \, q_0}$, i.e. the proportionate increase in money turnover, $\frac{\Sigma p_1 \, q_1}{\Sigma p_0 \, q_0}$, divided by the Paasche price index.[2]

2.2 The *price cuts* occurred every March 1st until 1952. From that year inclusive they occurred on April 1st. The change was doubtless made in order to impress the international economic conference that opened in Moscow on April 1st, 1952, but there are also obvious advantages in choosing a quarter day. The first cut of all, however, coincided with the currency reform (December 15th, 1947), and in 1948 there was no proper cut : only reductions on April 10th and in December so inconsiderable that they are not now counted in the series.[3] It is explained at the time of the cut in the leader columns of *Pravda* that the population saves so many rubles through them on a yearly basis. This figure is always ex ante : there is no ex post correction of it at the end of the year, but if occasion is had to refer to it in the following January the original figure is repeated (' the population *saved* so many rubles ').[4]

I assume here that the saving is calculated on the basis of the trade volume already planned for the last 9 or 10 months of the year, and then brought up to a yearly basis by a factor allowing for the other 3 or 2 months. This factor doubtless allows for the planned movement, quarter by quarter, of trade volume : for Soviet trade sources are most insistent on quarterly figures. We require, then, an adjustment for the decline in trade during the first quarter. Now that the price cuts are an established institution, we may be sure that purchases of durables are very low in the first two or three months of any year. The first quarter's turnover is normally planned to be 8–10 per cent below that of the previous fourth quarter.[5] Allowing for growth, this means that about 21 per cent of the calendar year's trade volume takes place in its first quarter, and 15 per cent in its first two months.

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[1] *Pravda* October 6th, 1952.

[2] N. Ryauzov and N. Titel'baum, *Statistika Sovetskoi Torgovli*, Gostorgizdat 1951, p. 191.

[3] Thus the *Pravda* leader of March 1st, 1950, calls the cut of March 1st, 1949, the second.

[4] With the exception of 1953 ; *Gosplan*, results for 1953.

[5] *Finansy i Kredit S.S.S.R.*, 3/1952.

In December 1947, March 1949 and March 1950 the population's ' economy ' on state trade was given separately from that for co-op. and Kolkhoz trade[1] : the cut in the last being merely the natural result of competition from state and co-op. trade (the government imposes no price controls on the Kolkhoz market). But since the volume index applies to state and co-op. trade we must sort out the co-op. from the Kolkhoz ' economy '. Kolkhoz trade was 12 per cent of all trade in the latter years (Appendix II), and therefore about half as much as co-op. trade. But both cuts weighed very heavily on food items, which bulk much larger in Kolkhoz trade. I have therefore alloted ½, not ⅔, of the co-op and Kolkhoz ' economy ' to the co-ops in 1949 and 1950. In 1947 the ' economy ' weighed very heavily indeed on Kolkhoz trade (it cut prices by 70 per cent)[2], so I allocate to the co-ops. only ⅓ of the combined co-op. and Kolkhoz cut. From 1951 the ' economy ' is given for state and co-op. trade together, so that it needs no adjustment.

The Soviet formula for the ' economy ' is $\Sigma p_0 q_1 - \Sigma p_1 q_1$. [3]

2.3 *The money value of retail goods turnover* is on the other hand usually a secret. Certainly it has never been directly revealed.[4] The figure of R. 306 md. for 1947 is arrived at thus. The planned turnover for that year (R. 325 md.) is, alone of all these figures, given straight, in the published extract from the 1947 plan. It should bear the same proportion to the actual figure as did anticipated to actual revenue from the turnover tax in that year. The 1944 figure is also firm, and we have for that year a turnover tax revenue in rather lower ratio to turnover. Interpolating arbitrarily between these ratios we obtain the 1945 and 1946 turnovers.

The figures for 1953 are arrived at as follows. The planned figure for the last three quarters of 1953 was R. 312 md.[5] The 1953 ' economy ' was R. 50 md., thus R. 39.2 md. in the last three quarters. Then at pre-cut prices turnover in the last three quarters would have been R 350.5 md., and in the first quarter about 21/79 of that, or R. 93.5 md. Then total planned trade = R. 405.5 md. Actual retail turnover in the last three quarters of 1953 exceeded the plan by R. 33 md.[6] I assume the first quarter's plan was, as generally while Stalin lived, under-fulfilled.[7] Put then 90 + 312 + 33 = R. 435 md. for the year's actual turnover.

The figures for 1948 to 1952 are also interpolated by means of the turnover tax, as follows : we have to move from a tax/trade ratio of 78.6 per cent (1947) to one of 60.1 per cent (1953). Large changes in the ratio are set against large ' economies,' and allowance is also made for the number of months

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[1] The three types of trade are distinguished and explained in Appendix II.

[2] *Pravda* April 15th, 1948. It will be appreciated that owing to excess demand Kolkhoz prices were far greater than state and co-op. prices until 1948. Thereafter the difference seems to have been negligible.

[3] Ryauzov and Titel'baum, *op. cit.* p. 181.

[4] Before the war it was always revealed, and the volume index never.

[5] *Gosplan* results for 1953.

[6] *Gosplan, ibidem.*

[7] Trade organisations fulfilled their turnover plan in 1953 ' for the first time in recent years ' (Mikoyan, *Pravda*, April 27th, 1954).

(seasonally weighted) over which the price cut ran in the calendar year. The turnover value is then deduced from the interpolated tax/trade ratio.[1]

2.4 *The official price index for state and co-op. trade* is not regularly published but occurs sporadically in the speeches of Party leaders. See the notes to Table II. The Paasche formula is used, and the excuse given for this is interesting: it embodies the two elements required to calculate the ' economy ' (2.2 above). It will be observed that the ' economy ' is simply the Paasche denominator minus the Paasche numerator. That this formula generally gives smaller rises and greater falls than the Laspeyre is not, of course, mentioned, at any rate in the only source available to me.[2] The index is superior to a Western one in that it embodies no sampling: prices everywhere are known since they are centrally controlled, and turnovers at current prices (not physical quantities) must be reported to head offices by all trading organisations. Hence the index is based on all reported turnover, thus:[3]

$$\frac{\Sigma p_1 q_1}{\Sigma \frac{p_0}{p_1} p_1 q_1}$$

Naturally one can deduce the official price index from the volume index and the money values of state and co-op. trade. This index has been calculated, as is the official, on the IV/1947 base. I have assumed that both volume and prices were higher in that quarter than in the rest of the year, the former because of the upward tread of volume and the Christmas peak, the latter because of the inflation that reigned until the monetary reform. Put, then, annual rates of 96 and 330 for the volume index and the money value respectively; the other years are obtained by dividing money value by volume index, with $\frac{330}{96} = 100$. The result is not so very different from the official claims, and shows that the Soviet volume figures are—as is only to be expected in so simple a tautology—roughly consistent with their claims as to prices.

Kolkhoz market prices are very volatile and differ in each town. An index is compiled for them out of a sample taken in very many different towns,[4] but it is rarely published.

Finally there is a general index of the cost of living, expressed inversely as an index of the purchasing power of the rouble. This includes all three types of trade, and also services (with a weight, in the example given, of 26 to 29 per cent).[5] The index is very rarely published, see 6.1 below.

[1] Between 1947 and 1953 the turnover could also be calculated from the ' economies ' by the following formula:

$$T_1 V_2 - E_2 m = T_2$$

where T_1, T_2 are the turnovers of two successive years, V_2 the official volume index connecting them, E_2 the ' economy ' in the later year and m the weight, allowing for the seasonal correction previously devised, of the months in the calendar year for which it ran. This procedure is not very successful. Chained, it produces a large cumulative error if we work back from 1953 and a very large one if we work forward from 1947. If we take pairs of years separately the results still usually diverge unsatisfactorily from those obtained by the turnover tax method. Evidently the actual, ex post, ' economies ' differ greatly from the planned, ex ante figures given.

[2] Ryauzov and Titel'baum, *op. cit.* p. 181.

[3] *Ibid.*, pp. 174, 183.

[4] *Ibid.* pp. 174, 186–8.

[5] *Ibid.*, p. 194.

3.1 *The factor cost* of producing the goods that enter into state and co-op. trade is their current money value less the turnover tax actually applicable to them. This is taken to be 63 per cent of all turnover tax in mobilisation years and 70 per cent in peace-time (Appendix I). There results an extremely approximate figure for factor cost, which, be it noted, is not necessarily a magnitude more closely representing true scarcity relationships than market price, as in the West. For while it may well do this in manufactured goods, in the all-important food items market price is probably a more reasonable measure of true scarcity relationships, since factor cost of most production is the price for compulsory deliveries from collective farms; a price so very low that the Soviet authorities themselves describe the whole procedure as ' in the nature of a tax.'

3.2 Having deduced factor cost we can express the *average rate of turnover tax* in the ordinary Western manner as a percentage on factor cost. When the figures are seen (Table II col. 11) it becomes clear why the Communists always express them as a percentage of gross retail price. For what in the one language is 300 per cent is merely 75 per cent in the other. This is doubtless also the main reason for concealing the money value of retail trade. Small errors have a large effect on these rate estimates. Thus if not 70 per cent but 63 per cent of all turnover tax were paid by state and co-op. trade the rate of tax in 1947 would fall from 122 per cent to 99 per cent. Individual rates of turnover tax are not known after the war, and the average rate may well conceal a multitude of interesting variations.

3.3 The lesson of these last three columns in Table II is, however, clear. The average rate of turnover tax was about twice its pre-war level in 1947, and is now about back to that level. No wonder we have no post-war data on the rate of tax. The reduction in costs has lagged behind the reduction in tax since 1947: a not surprising conclusion. Table II also fits quite convincingly with certain events. Thus on January 1st and July 1st, 1950, wholesale prices were lowered, and our ' unit factor cost ' shows a greater fall than in any other year. On January 1st, 1952, they were lowered again, and in 1952 also a great fall in unit factor cost took place. In 1953 the average rate of turnover tax fell more than in any other year : Stalin died in March, and numberless concessions were made to the consumer. The rise in ' factor cost ' between 1952 and 1953 may well be due to the rise in the prices of compulsory and other farm deliveries. On the other hand wholesale prices were very sharply raised on January 1st, 1949, and subsidies to industry were much lower in 1949 than in 1948. There should surely not be so great a fall in factor cost between these two years as the table shows, and there is something here to be cleared up.[1]

In 1953 and 1954 agricultural procurement prices (both compulsory and other) have been raised. As well as the lowering of retail prices this too will have lowered the rate of turnover tax.

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[1] Mr. Nove's index of investment prices gives a series roughly corresponding to factor cost for that lowly taxed sector of the economy. It moves in a much more convincing manner (*op. cit.*, p. 78).

3.4 In 1951, incidentally, the definition of retail trade was expanded to include: bespoke tailoring, shoe and watch repairs, etc.; the sale of livestock and produce by state farms and ' secondary farms '; the sale of houses and summer cottages.[1] I assume that all this comes to about 2 per cent of trade and that it is included in both the money total and the volume figures for 1951 onwards So long as we are only discussing the price index no adjustment is required. In general the Soviet concept of retail trade covers material products sold in shops. It excludes, then, entertainment, travel, gas and electricity. Supplies to the army, the concentration camps and their guards do not enter into retail trade. Those to restaurants do, and apparently also those to factory canteens.

4. We have now at last an official figure for the *money wage of workers and employees:* in 1952 it was 201 per cent of 1940.[2] There are, as we know,[3] two possible figures for 1940: R. 4069 and R. 4051 per annum. This gives R. 8163 for 1953, a figure slightly but not disconcertingly greater than that hitherto accepted by most Western statisticians. For 1948 I shall continue to take my own much less firmly based R. 7080.[3] As to the intervening years, for 1949 the *Gosplan* results made no mention of money wages: presumably, then, they did not rise in comparison with 1948. In 1950, 1951 and 1952 they were said to have risen by unspecified amounts; and in 1953 by 2 per cent over the previous year—a precision in keeping with the greater communicativeness that followed Stalin's death. A tentative series is set out in Table IV with the appropriate interpolations.

5.1 All estimates of real wages must henceforth be based on Mrs. Chapman's comprehensive *cost-of-living index*, compiled independently from actual price observations in four years.[4] This is much the most solid and complete index yet produced. Mrs. Chapman's work is so important that it is worth while to criticise its defects in some detail before they pass into general use and become canonical. Her figures are as follows (1937 = 100):

TABLE III

	1928	1948	1952
Official prices, all commodities, U.S.S.R.	11.5/15.4	316/316	206/202
Average prices all markets, all commodities and services, U.S.S.R.	14.3/20.2	307/315	207/202

The first figure in each pair is based on 1928 value weights and the second on 1937 value weights. The two indices agree admirably for the years that principally concern us.

However the divergence for 1928 is very great. The two weighting systems give respectively the Laspeyre and the Paasche formula for 1928–37. Now in ordinary cases the Paasche formula gives a lower result because there is substitution away from the goods and services whose prices rise most;

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[1] Ryauzov and Titel'baum, *op. cit.* pp. 72–3.

[2] In the new textbook of economics, *Politicheskaya Economia* (Moscow 1954), p. 452.

[3] Wiles, BULLETIN, September 1953, p. 333.

[4] Set out with much of its working in the *Review of Economics and Statistics*, May 1954.

consequently end-year weighting gives these prices less importance. But in U.S.S.R. 1928–37 the turnover tax was introduced and the whole price structure revolutionised. Necessities increased in price more than luxuries, bread leading, as Mrs. Chapman's sub-indices show. Also by any formula real wages fell, and the relative rise in the price of necessities was not so great as to make them impossible to buy. So superficially it would appear that they were inferior goods, absorbing a larger proportion of income in 1937 than in 1928. If this had been so we should expect the Paasche result to approach *or even to surpass* the Laspeyre result. But no positive evidence can be offered[1] that in a planned economy unresponsive to changes in demand the production of inferior goods actually expanded—or failed to contract—enough to enable consumers to spend a larger portion of their incomes upon them, at the prices fixed by the government.

These considerations lead, then, to no definite conclusion. It suffices, I think, to rule out the Paasche formula for 1928–37 on the simple ground that it will undoubtedly be compared with foreign cost-of-living indices all using Laspeyre or near-Laspeyre formulae. What is sauce for the goose is sauce for the gander, and we must take Mrs. Chapman's 1928-weighted, Laspeyre, result. This, in its turn, errs in that bread, which rose in price by much more than the average, is underweighted owing to the choice of a skilled worker's budget. However there are various compensating factors, too unimportant to mention individually, which offset this. We shall accept the index as it stands.

5.2 Mrs. Chapman's index of official commodity prices, which covers precisely state and co-op. trade, shows a fall of 35 per cent between 1948 and 1952. This is close to my own fall of 41 per cent deduced in Table II and not so very far from the official figures, which are again confirmed. The difference is just such as might be expected between two Paasche indices and one using a modified Laspeyre formula (Mrs. Chapman uses, it will be recalled, 1928 or 1937 weights for the 1948/52 comparison). Most unfortunately I can find no direct way of bringing Mrs. Chapman's 1937 into comparison with 1940, for, inflation having much increased, Kolkhoz market prices, level with others in 1937, much exceeded them in 1940. It is unfortunate that the Soviet authorities have chosen so awkward a base year, but it is certainly understandable, since real wages have seldom been lower in peace time than in 1940 (perhaps only during the famine of 1932–3).

But something must be done about 1940, since it is the base year for all modern Soviet statistics. Indirectly we may work back from the reasonably close correspondence between Mrs. Chapman's and my indices from 1948 and 1952. With these as a starting point we get an index of official commodity prices (1937 = 100) of about 131 in 1940. What was the excess of Kolkhoz market over official prices in 1940? Available data strikingly illustrate the elusiveness of truth in U.S.S.R. For ten similar commodities in Moscow in 1940, according to the U.S. Embassy report in May of that year, the gap was 77 per cent.[1] According to Mr. Schwarz prices rose throughout the year, and we may take it this report referred to the early months. According to Prof. Prokopovicz[2] the gap for ten commodities was 270 per cent (arithmetic mean). Omitting the very extreme figure for potatoes it was 125 per cent. According to 'unpublished sources, the authors of which prefer to remain anonymous', Messrs. Bergson and Heymann[3] make the gap 50 per cent. I shall make bold to put it at 100 per cent. Kolkhoz market prices were then 262 per cent of their level in 1937, when they did not differ from other prices. Their weight in a worker's or employee's budget might be about $\frac{1}{4}$.[4] The prices of services, rents, etc., we shall assume to have risen half as fast as those of commodities in official trade (they show themselves noticeably stickier in all Mrs. Chapman's indices), and to have a weight of 8 per cent (Mrs. Chapman's 1937 estimate). This yields a very tentative all-inclusive index of 162, 1937 = 100.

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[1] Thus it is not evidence to compare Mrs. Chapman's weights for 1928 and 1937, since her 1937 weights are derived from the total of goods entering into consumption and her 1928 weights from a sample of *skilled* workers' family budgets. Although she does not say the workers are skilled this can be proved from the suspiciously close correspondence of her weights to those given for skilled workers in S. P. Turin, *International Labour Review*, October 1929.

6.1 The real wages of workers and employees may then be set out as follows:

TABLE IV. *Prices and Wages*

	Retail price indices, all U.S.S.R.				*Money Wages*		
	State and co-op. trade only *Deduced*	*Official*	*Chapman*	*All goods and services (Chapman)*	*Rubles p.a.*	*As Index*	*Real wage Index*
1928 ...	—	—	11.5	14.2	726[3]	23	163
1937 ...	(77)[1]	—	100	100	3038[4]	100	100
1940 ...	100	100	(131)[1]	c. 162[1]	4061[4]	138	c. 81[1]
IV/1947 ...	261	278	—	—	—	—	—
1948 ...	251	—	316	311	7080	233	75
1949 ...	213	—	—	269[2]	7080[2]	233	87
1950 ...	173	—	—	230[2]	7390[2]	244	106
1951 ...	164	—	—	220[2]	7690[2]	254	116
1952 ...	149	—	204	204	7990[6]	263	129
1953 ...	141	143	—	194[5]	8163	269	139

Sources. Unless specified below: Chapman *op. cit.*; Table II above; and section 4 above.
[1] Peculiarly hazardous: explained in text. [2] Interpolated.
[3] Both 703 and 750 occur in official sources: Wiles, BULLETIN, Sept. 1953.
[4] Official. [5] Extrapolated.
[6] 2 per cent less than in 1953: *Gosplan* results for 1953.

The cost of living index for 1953, if expressed on the base 1940, is 120. This corresponds with a closeness my methods surely do not deserve to the very latest official release[5]: that the 1953 index for all goods and services, including the Kolkhoz market, is 122 on the base 1940. Admittedly the source does not tell us what months are referred to in either year (one of rising, the other of falling prices); also a Paasche formula will have been used, and a much greater weight taken for services than by Mrs. Chapman (see 2.4). But our doubtful 1940 figure receives needed confirmation.

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[1] S. Schwarz, *Labor in the Soviet Union*, p. 170.

[2] *Quarterly Bulletin of Soviet Russian Economics*, May 1941.

[3] *Soviet National Income and Product*, 1940–48, p. 227.

[4] N. S. Margolin, *Balans Denezhnykh Dokhodov Naselenia*, 1940 edition, pp. 114–121. This proportion is derived from his tables of the circulation of money in the economy, which he made, for security reasons, approximate only.

[5] *Textbook of Economics*, p. 462.

These figures omit the following items: the income tax, the state loan, the social services and the interest on the state loan. All these deductions and additions formed a larger percentage of income in 1940 than in 1928, and in 1952 than in 1940. Mrs. Chapman[1] finds that if they are all taken into account the net result is not widely different from the gross. However in 1953 the state loan was halved, and this should be remembered when projecting our series forward. Further there are certain other qualifications Mrs. Chapman omits. First and foremost are the new consumer goods which have come on to the market since 1928. All cost of living indices much underestimate the increased welfare due to such innovation, but probably nowhere so much as in U.S.S.R., since the proportion of new to old types of consumer goods in the period under review must have been peculiarly high. Secondly sales of own produce, sales of property, subletting and boarding, loans, credit and pawning amounted to 3.8 per cent of well-paid workers' incomes in 1927[2] and perhaps 1.6 per cent of all workers' and employees' incomes in 1940.[3] Again the size of family fell by perhaps 7 per cent between 1928 and 1938,[4] and must, if mortality claims are anything like correct, have greatly fallen again since that date.[5] Finally the number of incomes per family much increased between 1928 and 1937 owing to the absorption of women into industry, but against this must be set the decline of home amenities that it brought about. Since 1937, however, the concentration camp system has been in full swing and has combined with the last war to reduce almost unbelievably the proportion of live (let alone free) adult males to females. This must be presumed to have increased the proportion of separate households to earners, and represents a grave diminution even of merely economic welfare. We may dispute whether a wife at work is a favourable or an adverse economic phenomenon; but it is clearly a bad thing if millions of households are very small and lack a male head.[6]

6.3 With this we may turn to the Soviet claims as to real wages of 'workers and employees'. They are as follows:

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[1] *Op. cit.*, pp. 150, 146. [2] Turin, *op. cit.* [3] Margolin, *loc. cit.*

[4] F. Lorimer, *Population of the Soviet Union*, p. 226.

[5] It is exceedingly difficult to believe that a people about a third as well off as the British (Schulz and Wiles, BULLETIN, Sept. 1953 Table V) should have a lower mortality rate (8.9 per 1,000; Mikoyan, *Pravda*, April 27th, 1954). If this is so, however, the Soviet birth rate is phenomenally low: a mere 24 per 1,000 compared with 38 per 1,000 in 1938 (for the rate of natural increase is about 1.5 per cent p.a.). This drop is not improbable, however, in view of the extreme unbalance in the sex ratio discussed in the next footnote.

[6] I estimate (by processes too long to defend here) the surplus of free females aged 16 plus as follows:
1926 5 mn. (out of 47 mn. females free or in prison).
1939 12 mn. (out of 57 mn. females free or in prison).
1953 15–20 mn. (out of c. 70 mn. females free or in prison).
For those who prefer official data the percentage of female to all workers and employees was 37 per cent in 1939, 47 per cent in 1947; and to-day 60 per cent of all doctors are women.

TABLE V

Various Real Wage Series

	Derived from Official Sources			*Derived from Table IV*
		(chained)[1]	*(adjusted)*	
1940	100		100	100
1947		under 50[4]	under 55	
1948		100	109	93
1949	124[2]	112	122	107
1950		133.5	143	131
1951	157[3]	147	157	144
1952		157	168	160
1953		174	185	172

[1] *Gosplan* yearly results, chained.

[2] *Gosplan* results for 1949.

[3] Malenkov, *Pravda* October 6th, 1952.

[4] The Soviet concept is 'income in comparable prices' throughout, except that in 1948–47 it is 'real wages'. It should be borne in mind that in the crop-year 1946–7 there was a near-famine; the figure for 1947 is not, then, impossible, though the trick may have been played on us, not unknown in Soviet statistics, of giving the increase in the real wage bill, not real wages per head. The source is not clear on this point.

As so often, the later figure (Malenkov's 157) is lower than what would result from chaining the annual results. In interpolating these latter, then, I have made some arbitrary downward adjustments, but they are small and the internal consistency of these figures is not desperately bad. As can be seen, the series compares well with our own.

6.4 These figures may be checked in one further way, which is too hazardous to provide more than confirmation or reasonable doubt. The expenditure of workers and employees on state and co-op. trade was about 60 per cent of that trade in 1938–9, according to Margolin's rounded estimates.[1] We may therefore expect real wages to move in rough harmony with the volume of trade per worker and employee. Now the trade applicable to workers and employees is roughly that of all urban co-ops (see Appendix II), state trade and Kolkhoz market trade. Few purchases in these categories are made by country-dwellers[2] (e.g. the collective farmer who sells a chicken in the Moscow Kolkhoz market may buy a shirt in Gorki Street before going home), and few purchases in the remaining category, rural co-ops., are made by town-dwellers (e.g. the civil servant on summer holiday in his dacha buys his groceries at the local village co-op.). Table XI (Appendix II) shows that village co-ops. took up 26 per cent of state and co-operative trade in 1940, and 16 per cent in 1947 and 1950; also that Kolkhoz market trade *in real terms* (i.e. after allowing for inflated prices) was about 12 per cent of all trade throughout. The 'volume of urban trade' in Table VI is drawn up on the basis of these data.[3]

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[1] *Loc. cit.*

[2] A disturbing factor is employees of state farms and Machine Tractor Stations, who are both country-dwellers and workers and employees. These were only 6 per cent of all workers and employees in 1941 (1941 *Plan*, p. 512), and their number appears to have expanded, judging by the number of state farms and MTS, about as fast as those of other workers.

[3] Also of two smaller considerations assumed to offset each other: the expansion of the network of Kolkhoz markets means that more private trade is statistically recorded than in the past, and peasant purchases in towns must have diminished with the expansion of rural trade per rural head.

TABLE VI

Urban[1] Trade per head and real wages

	Urban population, millions	*Workers and employees, millions*	*Volume of urban trade*	*Urban trade per urban head*	*Real Wages*	
					(*official*)	(*derived from Table IV*)
1940 ...	61[2]	30.8	123	136	92	108
1948 ...	c.67[3]	34.2	100	100	100	100
1952 ...	79[2]	40.2	194[4]	167	154	172

[1] Including ' workers' settlements '.

[2] Malenkov, *Pravda*, August 9th, 1953. [3] Interpolated.

[4] Deducting 2 per cent for the new, wider definition of retail trade previously quoted (sec. 3.4).

' Reasonable doubt,' then, is unquestionably what emerges from this check. Workers and employees spent, as we have seen, about 60 per cent of their income on state and co-op. trade in 1940. In 1948 they must have been spending much less : about 40 per cent if Table VI is to make sense. Now the Kolkhoz market's share of trade was roughly constant (Appendix II). This implies that with lower real wages than in 1940, workers and employees spent much more of their incomes on housing and services—a very unlikely proposition. The discrepancy, it is to be noted, occurs between 1940 and 1948. It can be closed a very little by deducting rents, trade-union dues, savings and direct taxes from the real wage series. These rose slightly faster than other outlays, being 15.7 per cent of the outlays of *all* households in 1940 and 19.4 per cent in 1948.[1] Real wages available for spending on retail trade thus rose by 4 per cent less than gross real wages. But this is small consolation. The most probable explanation is that my figure for real wages in 1940 is too low, because the price index for that year—the most hazardous of all our hazardous figures—is too high.

6.5 Another check is to compare the volumes of retail trade with the claimed index of production of consumer goods. How may these two indices be expected to diverge ? First the latter index includes only processed food : e.g. bread, confectionery, sausages, drink. But the production of food of all kinds has increased less rapidly than that of other goods, except textiles ; so this is a handicap to the trade index, which is more heavily weighted with food. Since furthermore all outputs are valued gross, and not according to the value added by the factory, Ministry or other reporting unit, the consumer goods index is likely to underweight food and textiles relatively to hardware, as the intermediate materials for these latter pass through the hands of several industrial reporting units while in the case of food and textiles the intermediate materials go to compose the index of gross agricultural production. Now the weight of manufactured goods in retail trade has risen from 36.9 per cent in 1940 to 45.3 per cent in 1954[2] (doubtless at current prices). This is almost solely due to the expansion of consumers' durables

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[1] Bergson and Heymann, *op. cit.* pp. 20–21. The estimates are uncertain, and include all urban and rural households.

[2] Mikoyan, *Pravda*, October 25th, 1953.

(see Table VIII). It is therefore certain the consumer goods index has an upward bias owing to the double counting of those goods, the production of which has risen most. This bias operates both absolutely and relatively to trade volume. Thirdly the upward bias introduced by the peculiar system of valuation known as ' 1926–27 prices ' is not thought to have been very great since 1940[1]; but such as it is it applies only to the production index. There never were any *retail* ' 1926–27 prices ', and since the trade index was first constructed after the war it was begun on a more rational basis. Fourthly imports of consumer goods, negligible in 1940, have steadily increased, but are still only a very small fraction of all trade.[2] So far as this goes, however, it gives a relative upward bias to trade volume. Lastly consumer goods were being heavily stockpiled or diverted to the armed forces in 1940, but not in post-war years; this too would lead us to expect trade volume to grow faster.

On balance then we need not be greatly surprised to find a fairly close correspondence between the two series, with the consumer goods index rising somewhat faster. And this is precisely the case until 1953. In that year there was clearly destocking and an increase in imports, owing to Stalin's death, so that the trade index caught up.

TABLE VII

	1940	*1944*	*1948*	*1949*	*1950*	*1951*	*1952*	*1953*
Retail Trade Volume ...	100	—	75	90	117	134	149	179
Production of Consumer Goods[1]	100	54	99	107	123	143	158	176

[1] Malenkov, *Pravda*, October 6th, 1952; plan results for 1952 and 1953.

The same rough correspondence can be seen for the following individual figures (indices for 1950, on 1940 = 100):

TABLE VIII. *Production and Retail Trade*

	Sold[1]	*Produced*[2]
Butter	159	154
Margarine and other fats	167	163
Fish	151	(*caught*) 124
Sugar	133	122
Confectionery	134	124
All textiles	147	101[3]
Shoes	139	114[4]
Watches	330	281[5]
Radio receivers	600	541
Bicycles	290	241

[1] Lifits, *Voprosy Ekonomiki* 7/1951, p. 58.

[2] Compiled from the annual reports of the Economic Commission for Europe.

[3] Numbers of metres of cotton, wool and silk, added together.

[4] Numbers of leather and rubber shoes added together.

[5] Clocks and watches.

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[1] Thus in Table I the rate of divergence between the official and the Hodgman index of *total* industrial production is much smaller after 1940 than before it. On this subject generally see Kaser, *Economic Journal*, 1950 ; Gerschenkron, *Review of Economic Statistics*, November 1947 ; N. Jasny, *The Soviet Price System*, passim.

[2] They valued R.4 md. at the frontier in 1953 (Mikoyan, *Pravda*, October 25th, 1953). Whatever the relation between import prices and retail prices this is a negligible sum.

Now Polish textiles and Czech shoes were the largest foreign importations of consumer goods mentioned to me in Moscow: so this correspondence is again good, since it is only in textiles and shoes that the figures seriously diverge.

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APPENDIX I

TURNOVER TAX AND RETAIL TRADE

The percentage of turnover tax levied—at all stages of production—on goods entering into state and co-operative trade may be estimated as follows for 1940. A. K. Suchkov[1] gives the following table for 1939 (Column A only):

Table IX. *Origins of the Turnover Tax*

Commissariat	*per cent of total turnover tax paid* (*A*)	(*B*) *of which applicable to retail uses*
Meat and Milk	7.3	3.1
Food	29.7	17.8
Textiles	13.0	6.0
Light Industry	2.6	2.1
Procurements	34.4	29.6
Oil	8.0	4.3
Other (Coal, Machinery, Chemicals, Metals, Timber, Fishing, Producers' Co-operatives, etc.) ...	5.0	
	100.0	62.9

The products of all but the last two commissariats go exclusively into consumption. But ' non-retail uses,' chiefly stockpiling and the consumption of the forces, the concentration camps and their guards, form an immense proportion of total consumption, and pay turnover tax.[2] Thus according to Voznesensky[3] state and co-operative trade received only the following percentages of output:

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[1] *Dokhody Gosudarstvennovo Byudzheta S.S.S.R.*, p. 16.

[2] Evidence that the turnover tax is levied on the forces is given by N. Jasny, *The Soviet Price System*, p. 83, and by Bergson and Heymann, *op. cit.*, p. 137. The same is true of both ' corrective ' labourers and their guards, since the rations issued to each are accounted for at ordinary retail prices, as can be seen from the Ukhta-Pechora ration scale of May 27th, 1937 (Mora and Zwierniak, *La Giustizia Sovietica*, p. 110; U.S. Information Service, *Forced Labor in the Soviet Union*, p. 47; retail prices for that date are available in S. Schwarz and Prokopovicz, *opp. citt.*) It may well be that only the income in kind of soldiers, slaves and guards is a ' non-retail use ', while the ' NAAFI ' element of consumption, which these classes buy with their money earnings, is part of ordinary retail trade. Thus the long definition of retail trade in Ryauzov and Titel'baum (*op. cit.* pp. 70–75) makes it clear that the former could not be included, but leaves the latter quite vague. But either way our argument is not affected. Stockpiling in shops pays turnover tax (see the budget law in *Pravda* April 28th, 1954), but additions to centralised state reserves are one of the very few uses that do not, Mr. Nove informs me. For this reason our figure for the Commissariat of Procurements may be a little too low.

[3] *Voyennaya Ekonomika S.S.S.R.*, translated by Public Affairs Press, Washington, p. 75.

TABLE X

	1940	1942
Flour and grits	86	73
Meat	43	23
Cotton cloth	46	9
Leather footwear	79	27

The decline in 1942 is attributed to 'the needs of the Soviet Army' and 'consumption by war industry'. This latter phrase is peculiar, and seems not to refer to the canteens of armaments factories, which are included in trade statistics. Be that as it may, I have applied the 1940 percentages to the relevant commissariats in Table IX, column B, to obtain the percentage of all turnover tax paid by them on their output entering state and co-op. retail trade. The 79 per cent for leather footwear has been generalised for 'Light Industry', and the commissariat of Food, which baked most of the bread but also produced all the jam, spirits, etc., has been given 60 per cent.

As to 'oil' and 'other', oil is used mainly in transport and agriculture, and of the 'others' only machinery is pure investment while fishing is pure consumption. We shall not go far wrong in allocating this 13 per cent of all turnover tax to state and co-op. trade and to other uses in the proportion one to two.

The resulting figure of 63 per cent is surprisingly low. But 1940 was a year of re-armament and military stockpiling; hence the inflation and the fall in real wages. To the other years I apply, quite arbitrarily, 70 per cent. For comparison, Bergson and Heymann allocate 79 per cent of all turnover tax to state and co-op. trade in 1937, 80 per cent in 1940 and 77 per cent in 1948.[1] But these figures are residuals, and take no account of Voznesensky's data on non-retail uses, which are all we have to go by. However hazardous, then, the above calculation seems a little more reliable. Prokopovicz[2] appears to believe that the whole tax falls on state and co-op. trade.

In 1949 the levy (it used to be at 0.5 %) on producer goods other than oil ceased.[3] It might have accounted for half of the item 'other' in Table IX. Much of this entered indirectly into trade, and the percentage of the whole tax involved is inconsiderable even without that. This change, then, is neglected.

APPENDIX II

THE RELATIVE WEIGHTS OF THE THREE TYPES OF TRADE

State trade, in the nationalised shops, serves the towns with every type of good, and *Co-op. trade*, in what are formally consumers' co-operatives, serves the villages, mainly with manufactures (on state and collective farms most food is home-produced). In the *Kolkhoz market* (which is situated in a town) the collective farm organisation itself sells the surplus foodstuffs of the collectivised farm land, and the individual members sell any surplus of their own produce from their private plots, and of what they receive as dividends in kind from the collective farm. Here too are sold second-hand goods of all sorts, the products of spare time craftsmanship, and the surplus food off the allotments of urban and state farm workers. The Kolkhoz market began in the early 'thirties as a quasi-legal black market. As neither buyers for sellers could live without it it was legalised. But it still resembles a black market in being wholly free, paying no taxes and subjected to no price control. Hence when there is no inflation its prices must meet state and co-op. competition; and any divergence of prices is a certain sign of inflation. Since the sellers formally own the products they sell it is a capitalist market. It is therefore not permitted to be a wholesaler or to sell on commission, for that would promote a revival of private capital. This heinous offence is severely punished as ' speculation.' Consequently each peasant must come in to sell his own produce, and the amount of labour used in distribution is astronomical. From 1946 to 1949 an attempt was made to relieve this, by *urban co-ops.*: the same co-op. organisation that sold manufactured goods to collective farmers was also permitted to buy food from them, and dispose of it in the towns in its own retail outlets. Townsfolk did not thereby become members of the co-ops. In August 1949 urban co-ops. were brusquely and silently liquidated.[1] We do not know why: it might have something to do with Voznesensky's fall; or the contemporary leftward swing in agriculture, which involved hostility to private plots, may have carried with it a hostility to distributive methods making private plots more profitable. After Stalin's death Malenkov and Mikoyan—both known as right-wing—have promised a revival of urban co-ops. or something like them,[2] and this has found statutory expression recently.[3] Finally from 1944 to 1947 the state ran *commercial* shops. These participated in the then much inflated profits of the Kolkhoz market by selling goods off the ration at prices approximately as high. ' Commercial ' turnover is included in state turnover.

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[1] Bergson and Heymann, *op. cit.*, pp. 218–9; Bergson, *Soviet National Income and Product in* 1937, pp. 136–7.

[2] *Op. cit.*, X/2.

[3] F. D. Holzman, University of Washington, unpublished.

We have the following figures for the proportion of trade falling under each type, at current prices. Note that Kolkhoz market prices were higher than the others from 1940 to 1947, inclusive, and this swells the proportion taken by that market in those years.

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[1] On August 24th, 1949, *Pravda* attacked all trading organisations in a leader but the co-ops. more especially: and said that from now on the state must supply the towns and the co-ops. the countryside. Cf. H. Schwartz, *Russia's Soviet Economy*, pp. 366–7.

[2] Malenkov, *Pravda* August 10th, 1953: Mikoyan, *Pravda* April 27th, 1954. Note that since Mr. Mikoyan's ministry was also attacked in the *Pravda* leader of August 24th, 1949, it can hardly have been his jealousy that suppressed the urban co-ops.

[3] In the ' Vegetable Decree ', *Pravda*, Sept. 29th, 1953, sec. 4.

TABLE XI

Relative Proportions of the Three Types of Trade

	As per cent of Total Turnover			*As per cent of State and Co-op. Turnover*	
	State	*Co-op.*	*Kolkhoz*	*All Co-op.*	*Urban Co-op.*
1937[1] ...	65.7	21.6	12.7	24.7	
1940[1] ...	59.3	21.6	19.1	26	
1944				26[8]	
1945				20[9]	
1946				18[9]	
1947 (plan) ...				22.5[10]	5.4[10]
1947				23[9]	7.3[11]
early 1948		22[5]			
early 1949[2] ...	67	21	12	24	
1950	c.74[4]	c.14	12[6]	c.16	
1952			14.4[7]		
1954[3]	64.2	25.4	10.4	28.3	

[1] Prokopovicz, *op.cit.*, p. 461.

[2] Co-op. and Kolkhoz are 33 per cent of the total: *Pravda* March 1st, 1949. Kolkhoz alone is 12 per cent: Lifits and others, *Ekonomika Sovetskoi Torgovli*, p. 317.

[3] *Textbook of Economics*, pp. 510–2.

[4] Lifits and others, *op. cit.*, ch. 9 (' nearly three quarters ').

[5] A. Klimov, *Voprosy Ekonomiki*, 6/1948, pp. 99–100, says: ' The retail trade turnover (*sc.* of consumers' co-operatives) rose to R. 70.4 md. in 1947, being one and a half times the 1946 level and twice the 1945 level . . . Consumers' co-operation takes up, in the post-war period too, a large proportion of soviet (sovyetskoi) trade: its share is about 22 per cent of the turnover or the country (strany) '. This is the record of a speech made in September 1948, and the 22 per cent seems to refer to some date in 1948. Clearly 22 per cent of all trade—state, co-operative and Kolkhoz—is meant, not of state and co-operative trade alone. Klimov's figure has been wrongly interpreted as meaning the latter by Prokopovicz.

[6] Orlov, *Voprosy Ekonomiki*, 8/1953, p. 129.

[7] *Sovetskaya Torgovlya*, 2/1954, p. 3.

[8] N. Ryauzov and N. Titel'baum, *Kurs Torgovoi Statistiki*, *Gosplanizdat*, 1947, pp. 64–5.

[9] Deduced from Table II and footnote 5 above.

[10] 1947 Plan: *Pravda* March 1st, 1947.

[11] R. 22.4 md.: *Gosplan*, results for 1947. Percentage deduced from Table II. There is also R. 19 md. for ' 1947 and the first five months of 1948 ': Khokhlov, *Pravda* July 3rd, 1948. This contradicts both the planned and the actual figures for 1947 so violently that it must be a mistake. Perhaps the first five months of 1948 only are meant.

ON THE DIFFICULTIES OF MEASURING WAGE DIFFERENTIALS

Since the first World War, most industries have developed national wage structures—that is, nationally-agreed systems under which it is intended that some workers be paid more than others, the prescribed differences being related to skill and responsibility, sex and age, type and intensity of work, and the time and (sometimes) the place at which the work is done. Most of these differences, which are the products of economic and social history, share the fate of other historical monuments: many are jealously preserved, some are restored or adapted, some are allowed to crumble away under the attrition of time, and a few—very few—are deliberately levelled out of existence. Every now and then, as with other antiquities, people ask whether they still serve their earlier purposes effectively: in this case, whether wage differentials still provide workers with the necessary incentives to acquire skill, to undertake responsibility, to work harder or longer, to change their jobs, and so on. And even those who are sceptical about their effectiveness in bringing about desired ends find it difficult to believe that they have no effect at all: in trade union negotiations and in the Press, for example, there is often considerable argument about the presumed effects of wage differentials on occupational shifts within or between industries. But as such arguments, when they rise above pure speculation, are usually based upon limited data, it seems worth while examining the kind of measures which are commonly used and considering how much meaning can be attached to them—and, incidentally, enquiring how far other measures are likely to modify the conventional picture.

(1) *Relationships between nationally-agreed wage rates*

The commonest, and simplest, way of measuring wage differentials is to take the money difference or the percentage ratio between the nationally-agreed wage rates of two characteristic categories of workers—for example, fitters and labourers in engineering, or engine drivers and porters on the railways. Before the first world war there was a tendency in most industries for percentage differentials to be maintained, but after it, as national negotiation became the rule, the simpler method of changing the wages of all categories by equal money amounts spread widely. Thus money differences tended to remain constant over quite long periods while percentage relationships fluctuated—usually in relation to the cost of living, as many workers' wages were formally or informally linked to this index. But as wages have been rising steadily since 1939, the consequent narrowing of percentage differentials between skilled and unskilled has forced itself on the attention of negotiating bodies and is now beginning to provoke pressure for maintaining or widening them.

But differentials based on selected pairs of rates are of limited significance.

In some cases no single pair of rates is sufficiently representative, and it may therefore be necessary to take several pairs, or to construct weighted averages of rates between which to measure differentials. Moreover, there are considerable numbers of those whose wages are not settled nationally—in particular, among the increasingly important ' semi-skilled ' group. It is true that the wages of many of these probably bear some agreed relation, in practice, to those of the skilled or the unskilled; but, as the precise relation is usually determined in the light of local circumstances, they are automatically disqualified for inclusion in any wage comparison confined to nationally-negotiated rates. In any case, wage rates may be bad indicators of the actual wages received and the differentials prescribed or implied by national agreements may be very different from those apparent in actual earnings. This is particularly true of engineering, where rates are minima and earnings vary greatly within occupations according to the district and section of industry and—perhaps most of all—according to the particular firm in which people happen to be working. Some of the excess of earnings over nationally-negotiated rates can be accounted for by the operation of higher rates agreed on a district basis, but much of it represents special additions made by particular firms in the form of bonus, merit, or lieu payments. Moreover, as these tend to accrue to the skilled (and semi-skilled) more than to the unskilled, differentials based on averages of earnings are often wider than those based on rates. Again, nationally-agreed rates give virtually no clue to pieceworkers' earnings. National agreements in engineering, as in many other industries, do specify a percentage by which pieceworkers' expected minimum earnings should exceed time rates; but in practice the range of pieceworkers' earnings is very much wider than of timeworkers', and the Minimum Piecework Standard or its equivalent is at least as poor a guide to the actual levels of earnings for pieceworkers as is the time rate for timeworkers. And where, as is not uncommon, the proportion of pieceworkers is higher among the skilled than the unskilled, the fact that pieceworkers earn, by and large, more than timeworkers again tends to produce wider skill differentials in average earnings than in rates.

Engineering, although it comprises a very important part of manufacturing industry, is of course an extreme case; and there are many other industries where the nationally-negotiated rate is not a minimum but a standard rate with virtually no local or special additions, and where there is little or no piecework. But here too earnings may appreciably exceed rates owing to the incidence of shiftwork, spreadovers, overtime, and so on. Although special terms of payment for these, if they arise, are specified in national agreements, the extent to which they arise naturally cannot be rigorously agreed in advance as it depends on the organisation of work and the staffing policy which is adopted in the light of the industry's (or the firm's) requirements and the employment situation in which it finds itself. Thus, even if wage rates remain unaltered, the effects of a changing organisation of work may affect differentials in actual earnings. The incidence of overtime, in particular, tends to be uneven between skill grades; and the tendency of the unskilled to work longer hours will narrow any skill differential based on average weekly earnings (and probably on 'Ministry-of-Labour-type'[1] average hourly earnings too). There are, in effect, several forces pulling in different directions which distort the pattern of differentials set by relative wage rates.

Most of the sources of possible discrepancy between rates and actual earnings, and between the differentials based on each, have grown haphazardly with the passage of time. Fifty years ago there was less payment by results and fewer special payments; and actual earnings bore a much closer relation to wage rates—although the latter themselves were then less uniform because they were settled on a district basis and not nationally. As we have seen, the almost universal tendency has been to increase the rates of the lower-paid proportionately to those of the higher by giving them equal (or sometimes actually greater) money advances. The effect of this in paypacket terms has to some extent been offset in manufacturing industry by the widening of the gap between rates and actual earnings to the advantage of the more skilled; but against this we have in turn to set the progressive shortening of the standard week—that is, the progressive lengthening of potential overtime hours—to the advantage of the unskilled on whom overtime tends to fall most. In no specific case can it be assumed that these contrary pressures on the earnings of workers of different grades of skill will cancel each other out, and that differentials in wage rates necessarily hold good of paypacket relationships.[2] Even where, on average, they do appear to hold good, this is often the result of a fluke of grouping or selection, And what is true of skill differentials is, *mutatis mutandis*, true of most other differentials too.

This is not to deny the importance of the wage rate—least of all of the standard rate. The minimum rate also has great practical importance, if only because it provides a floor for some workers' earnings even though the sky may be the limit for others'. But the relationship between selected pairs of rates has a rather different and more nebulous significance. In wage negotiations, 'League Tables' of wage rates payable to 'comparable' workers in different industries are often debated on the plane of sociology rather than of economics, and the impression is given that the wage rate has an importance as a fixed status-label for the worker irrespective of the contents of his paypacket—which may fluctuate considerably from week to week—or even of the average level of his earnings. This impression is even stronger in the case of skill and other differentials based on rates. Thus the present status of a skilled worker in many industries is expressed by a wage rate about 15 to 20 per cent higher than that of an unskilled man, and the status of a semi-skilled worker by a rate (if he has one) 5 to 10 per cent higher; but in practice these differentials may easily be doubled in earnings—or sometimes inverted. It is therefore surely useless to speculate on the possible effects of wage differentials until we can get a fuller and more realistic picture of them than can be obtained from wage rates alone.

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[1] *I.e.* total earnings divided by total hours, without allowance for overtime premia.

[2] In terms of net earnings—and since the introduction of P.A.Y.E. many workers may have come to think of their paypackets in this way—the skill differential will, on balance, tend to be rather narrower than would appear from figures of gross earnings.

(2) *Relationships between averages of earnings*

In the present series of articles on wages, we have frequently used the differentials between the rates of different categories of workers, as prescribed or implied by national agreements, as a kind of yardstick against which to set the corresponding differentials based on the average earnings of these categories. We have been interested in securing as fine breakdowns of earnings as possible, in order to ensure the greatest possible degree of comparability between the categories considered. For example, when examining a skill differential it is advisable to consider timeworkers and pieceworkers separately, and possibly on occasion to confine one's attention to workers in a single district, section of industry, or firm, so that the distortion of skill differentials due to the effect of factors largely unrelated to skill may as far as possible be eliminated from the start. For similar reasons—and despite the limited association of hours worked with grade of skill—we have often preferred to give differentials based on averages of hourly rather than weekly earnings.

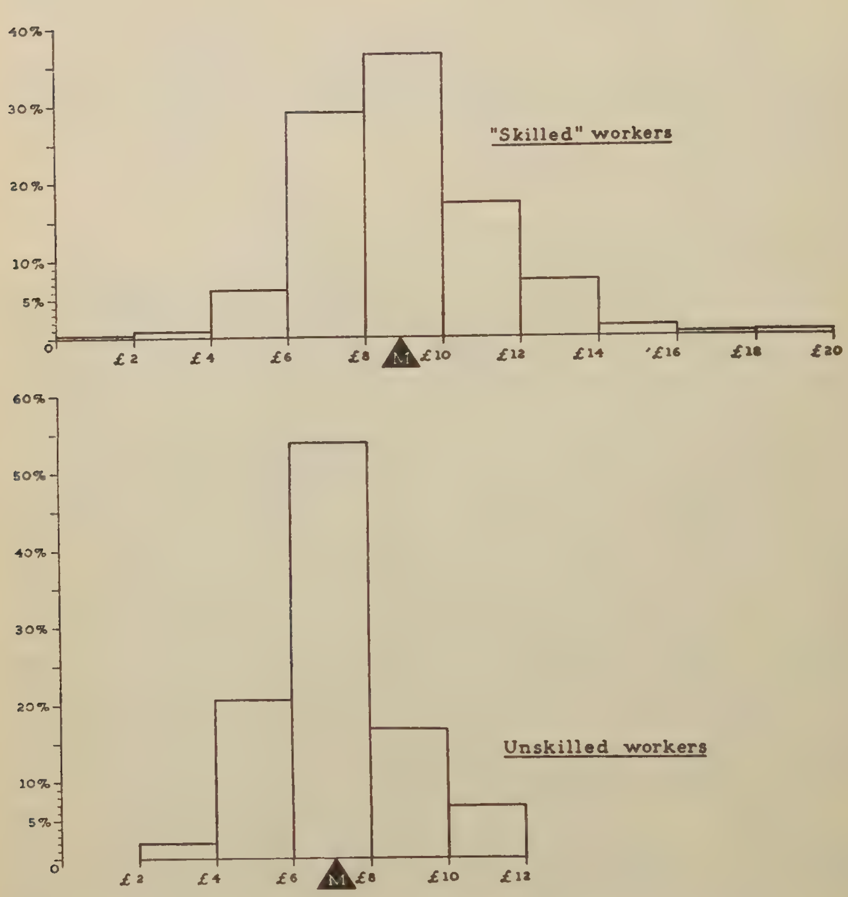
However, such a procedure has its disadvantages. For the finer the breakdown and the more homogeneous the categories which are being compared, the more unrepresentative the comparison will be for the industry as a whole—and therefore the less relevant the findings will be to the practical purposes of wage negotiation. If on the other hand our intention is not so much practical as theoretical, and we are concerned to isolate the effect of skill, then we have to admit that no breakdown however fine can possibly exclude all extraneous influences on earnings. In any case the form in which the material is available does not generally lend itself to detailed analysis. Most wage statistics (*e.g.* those assembled by the Ministry of Labour for its bi-annual surveys, and by many employers' associations for their own purposes) are collected in the form of totals of earnings, hours worked, and workers employed; and from these it is possible to calculate only a series of averages. The delimitation of frontiers between the groups for which the separate totals are collected conforms to standard patterns and is partly determined by the needs of administrative or practical convenience; and it inevitably gives rise to anomalies of classification and 'frontier incidents' of all kinds which, when attempts are made to analyse the figures or even to study the relationships between them, produce their own special sources of confusion. A more fundamental trouble is that averages are summary statistics, and any relationship based on them conveys at best only a very rough impression—and at worst a false one—of the relative levels of earnings of the majority of workers in the categories considered. For averages can take no account either of the great variability of individual earnings within single categories or of the frequent abnormalities in the shapes of earnings distributions. It is of course where two distributions are dissimilar that a differential based on their averages will be most misleading; and in most cases of this kind, while it can often be inferred from the evidence that the distributions must in fact be dissimilar, no reliable measurement of differentials is possible because the actual distributions are not available. It is, in short, only where we have distributions of workers' earnings that, whatever our purpose, we can hope to obtain adequate measures of wage differentials—*i.e.* reliable and informative indicators of relative levels of wages.

(3) *Other relationships*

Given such distributions, we are in a position to illustrate some of the limitations of conventional procedures in the course of suggesting one or two less familiar ways of measuring wage differentials. The examples which we are taking will be based on a pair of distributions of earnings—one for skilled and semi-skilled workers together and one for unskilled—derived from the national survey of personal incomes and savings carried out by the Institute of Statistics and relating to the financial year 1952–53. The original figures are a sample, drawn from the entire population, of the earnings of adult (18 years and over) male manual workers who were, during the year in question, employed on a full-time basis for at least 46 weeks. The distributions are of great intrinsic interest, both because of their coverage and because they relate not to a single week, as do most wage statistics, but to a whole year's earnings—although we have, for convenience, reduced the annual totals to weekly averages by dividing them by 50 (our estimate of the average number of weeks, including holidays, for which these workers were paid). All skill classifications are bound to be somewhat arbitrary, and in so wide a field an attempt to secure complete homogeneity would entail innumerable gradations with a consequent loss of generality. But unskilled jobs, for which virtually no training, aptitude, or experience is needed, can be distinguished fairly confidently from the rest—which for brevity we are lumping together under the heading of ' skilled ', further breakdown being impossible as degrees of skill cannot in practice be equated across industries. In order to avoid spurious accuracy—for distributions based on samples are naturally liable to error—the earnings have been grouped into broad intervals of £2. The basic distributions are shown in Diagram I.

However, before we attempt to compare relative wage levels on the basis of these data, there are certain characteristic features of such distributions which we must discuss. In the first place, the variation of earnings within each distribution is, obviously, due partly to variation in degree of skill. Accordingly, the variation in possible skill differentials—that is, in the extent to which the earnings of a skilled man differ from those of an unskilled man—is partly attributable to this factor. But its importance

I. Distributions of earnings (1952-53)



Note.

(*a*) In this and other diagrams, the black triangle indicates the arithmetic mean.

(*b*) The small proportion of "skilled" workers with extremely low earnings are apprentices. (For a general description of the data, see text.)

must not be exaggerated; for the distributions overlap considerably—thus for example nearly a quarter of the unskilled were earning more than £8 a week while over a third of the skilled earned less than this amount. But if variations in earnings were primarily due to different degrees of skill, the overlap and consequent inversion of the skill differential could not possibly be so extensive. From this alone it is obvious that there are other important determinants of earnings whose net effect is blurring the influence of skill. Nor is the general picture peculiar to these distributions because they happen to cover workers in all industries and of varying degrees of skill; for even within single and narrowly defined occupations we have found earnings—even district averages of earnings—ranging just as widely, and the earnings distributions for different grades of skill greatly overlapping each other. At any rate, we have reason to believe that the distributions we are using are in no way exceptional either in their range, their dispersion, or the extent to which they overlap. Indeed, in the present context the only peculiarity of earnings in these broad skill groups is that they are probably more normally distributed than those in many more homogeneous categories.

What is the relevance of such distributions to the concept of the skill differential ? If we were to select pairs of workers at random, one from each distribution, we should obtain a great variety of differences in earnings. These differences, as we have seen, relate partly to differences in relative skills but partly also to extraneous factors unrelated to skill which may vary widely from worker to worker, particularly—in manufacturing industry at any rate—between workers in different firms. For after all, many determinants of earnings are common to all workers in a particular firm : such factors as location, present and expected profits, and the ratio of labour to total costs, play an important part in shaping the firm's general wage policy; and glaring anomalies in earnings persist between one firm and another as a result of the notorious imperfections of the labour market. Or should we assume that a firm which pays high wages all round is necessarily blessed with a more skilled type of labour ? After all, it may be paying ' needlessly ' good wages for reasons of goodwill or prestige—or even careless rate-fixing. Again, the relative remuneration of workers of different grades of skill is itself extremely variable between firms. It may vary according to the composition of the firms' labour force : for example, a firm employing very little unskilled labour may be inclined to treat its labourers generously. It is by no means uncommon for the less skilled to be earning more than the skilled, either for the reasons stated or because the less skilled are working longer hours or are receiving special payments for more arduous, unpleasant, or intensive work. Similar payments are often made for other factors (age, length of service, good attendance or timekeeping, co-operativeness, etc.) which may or may not be particularly associated with skill.

Thus ' the skill differential ' in practice—*i.e.* the difference in earnings between skilled and unskilled—reflects the net effect of a conglomeration of factors of varying importance, of which skill is only one. In consequence such a ' differential ' does not, even on average, necessarily reflect the influence of skill on earnings at all accurately. It is not that isolating this influence would be statistically impossible, given adequate data. We could, probably, largely disentangle the effect of skill from that of other factors by variance analysis if, for example, details were available—say on a sample basis—of the earnings of individual workers simultaneously classified by occupation and by each of the other factors (location, industry, hours worked, size of firm, etc.). However, in default of this statistical Utopia (or purgatory?)

we have in the last resort simply to speculate on the effect of skill as against the net effect of everything else whenever we try to explain the great variety of wage differences between individual skilled and unskilled workers—or even between averages of earnings for these categories. However, at present we are not concerned with explanation (*i.e.* with assessing the relative importance of the various influences on earnings), but with the prior question of mere measurement. For the very variety of wage differences poses a real problem, if we are to obtain what we have called ' reliable and informative indicators of relative levels of wages '.

(*a*) *Measurements based on different types of ' average '*

Even if we follow conventional practice and measure earnings differentials merely by comparing summary statistics, the existence of distributions already provides us with a choice of three obvious measures: those based on the means, the medians, and the modes. Unlike many earnings distributions of narrower scope, the two we are using happen to be fairly symmetrical, and there is therefore not much to choose between the three measures (Table I). Even in this case, however, there is a little divergence: for the difference between the means tends slightly to exaggerate the relative advantage of the skilled—*i.e.* to overstate the gap in earnings which is most characteristic for workers in these categories. If however one of the distributions was violently skewed while the other was symmetrical (in shoemaking, for example, we found that the distribution of dayworkers' contract rates was positively J-shaped because large numbers were on or near the minimum rate, whereas that of pieceworkers' earnings was fairly normal), very different results would inevitably be obtained from each of the above measures. This point is by no means a purely captious one, as there is little reason to suppose that, in practice, distributions of skilled workers' earnings will necessarily be similar to those of unskilled.

TABLE I

Skill Differentials based on Different Types of ' Average '

	Means	*Medians*	*Modes*
Difference between skilled and unskilled ' averages ' ...	36/–	35/–	32/–
Skilled ' average ' as percentage of unskilled	126	125	123
Unskilled ' average ' as percentage of skilled	79	80	81

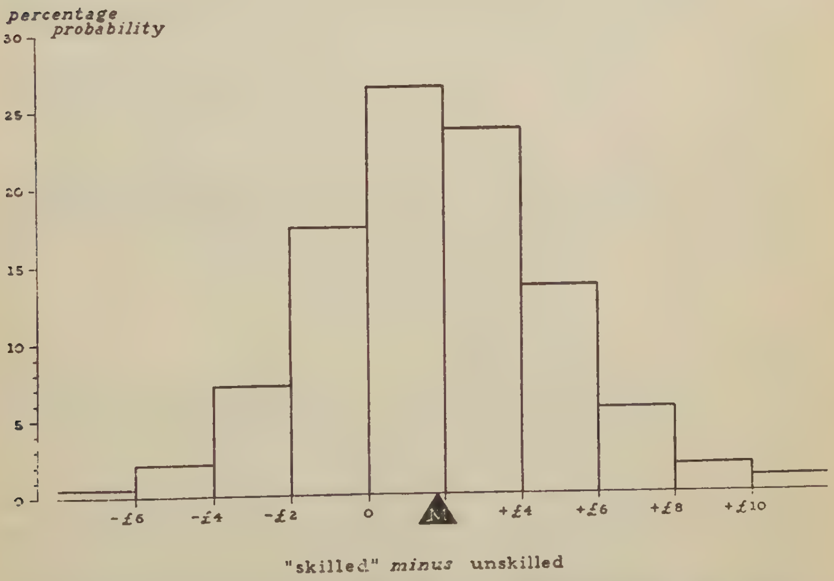
(*b*) *Measurements based on distributions of differences in earnings*

(i) *Money differences*

However, full distributions give us the opportunity of examining relative wage levels much more systematically than by looking at differences between averages. For it is obvious that there is a very wide range of differences between the earnings of different pairs of workers, and it is this universe of differences which is, in reality, ' the skill differential ' in earnings. How can we present this universe intelligibly ? What sort of distribution would we get if we were to tabulate the differences in earnings between all possible pairs of skilled and unskilled workers ? One important property of such a distribution is that its mean will be identical with the difference between the means of the two original distributions from which our pairs of workers were taken. The same is true, in effect, if we select pairs of workers at random, since the ' expected ' value (*i.e.* the mean) of the difference of two random variables is equal to the difference between their expected values. Thus when we take the money difference between the means of skilled and unskilled earnings—the simplest imaginable measure of the differential—we are unwittingly toying with a rather more sophisticated statistic, namely, the expected value in the probability sense of the difference in earnings between skilled and unskilled workers.

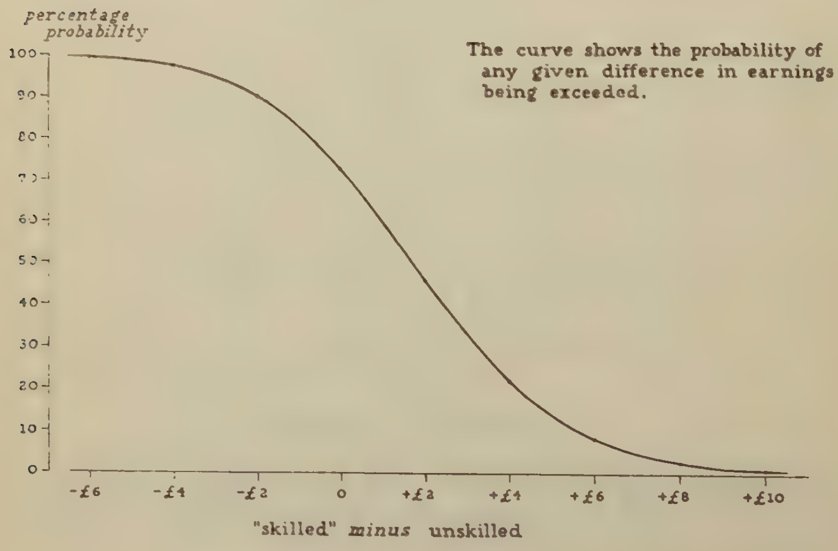
The entire frequency distribution of the differences in earnings between all possible pairs of workers selected from the original distributions of Diagram I has been obtained by the method described in the appendix, and is shown in Diagram II. As the frequencies in the distribution are presented in the form of percentages of the total frequency, they can be interpreted as

II. Probability distribution of differences in earnings



showing the probability that the difference in earnings between a skilled and an unskilled worker selected at random will fall in particular intervals—the intervals being of £2 as in the original distributions. The mean of the distribution will, as we know, be the same as the relevant figure in Table I (36s.). The median is approximately 34s., which indicates that if we select pairs of workers at random we expect that, in half the cases, the skilled worker will earn at least this amount more than the unskilled worker. The most likely difference in earnings, which is given by the mode of the distribution, is about 30s. The dispersion of this distribution is naturally much greater than that of either of the original distributions. The inter-quartile range is a little over £4, the quartiles themselves being about 75s. and minus 6s.; which means that in a quarter of the pairs we can expect the skilled worker to be earning at least 75s. more than the unskilled worker, while in a further quarter we anticipate that the skilled will be actually earning at least 6s. *less* than the unskilled. The latter is, of course, not surprising, for it was obvious from the overlap of the original distributions that the commonly assumed superiority in earnings of the skilled over the unskilled would be reversed fairly often in practice. Since, as we have seen, the skill differential is thus inverted in just over a quarter of possible cases, we can say that the chance of a skilled worker earning more than an unskilled worker is rather less than 3 to 1. If we wish to pursue this sort of approach, we can find the probability of a skilled worker earning at least any specified amount more (or less) than an unskilled worker by cumulating the probability distribution shown in Diagram II. This is done in Diagram III, which tells us that—for example—the probability of a skilled worker earning at least £6 more than an unskilled is about 8½ per cent: that is, the odds

III. Cumulative probability of differences in earnings



against are 11 to 1. Similarly, the probability of his earning at least £2 *less* is about 10 per cent, or 9 to 1 against.

This procedure already gives us a more realistic picture of the skill differential if only because it stresses the element of chance in human affairs. For it is partly by chance that people happen to work in well- or ill-paid sections of industry, or districts, or firms; and in consequence the accepted wage relationship between the skilled and the unskilled in the working population as a whole will in most cases be exaggerated, reduced, or inverted.

(ii) *Percentage ratios*

It is, however, often more instructive to measure differentials in earnings by percentages rather than by money amounts, if only because a percentage ratio is the better indicator, during periods of rising or falling prices, of relative wage changes in real terms. Our next problem is therefore to obtain the ' universe ' of all possible percentage ratios corresponding to that of money differences shown in Diagram II. This diagram throws no light on percentage differentials because the percentage value of any particular money difference depends, of course, on the figure of which it is a percentage; so that we have to go back to the original distributions and transform them to a logarithmic basis before we can get distributions of all possible percentage ratios—which we do by basically the same method as we used to obtained distributions of money differences (see appendix). It will already have been noted that in Table I we showed percentage ratios in each of the two possible ways (skilled as percentage of unskilled, and *vice versa*); and in Diagram IV we give the two corresponding distributions of percentage ratios.

The averages of each of these distributions are shown in Table II. It will be seen that the arithmetic means of all possible ratios are 131 (skilled as percentage of unskilled) and 88 (unskilled as percentage of skilled). The most likely differentials (*i.e.* the modes) are considerably lower than these, although in the upper distribution there is a wide range within which this value may fall—in fact, a skilled man is ' most likely ' to earn almost anything between 0 and 20 per cent more than an unskilled man. The medians fall about halfway between the other measures, and there are even chances of a differential falling above or below these levels. The interquartile ranges are 57 for the upper distribution and 38 for the lower. In the former case the quartiles themselves are 155 and 98, so that there is an even chance that a

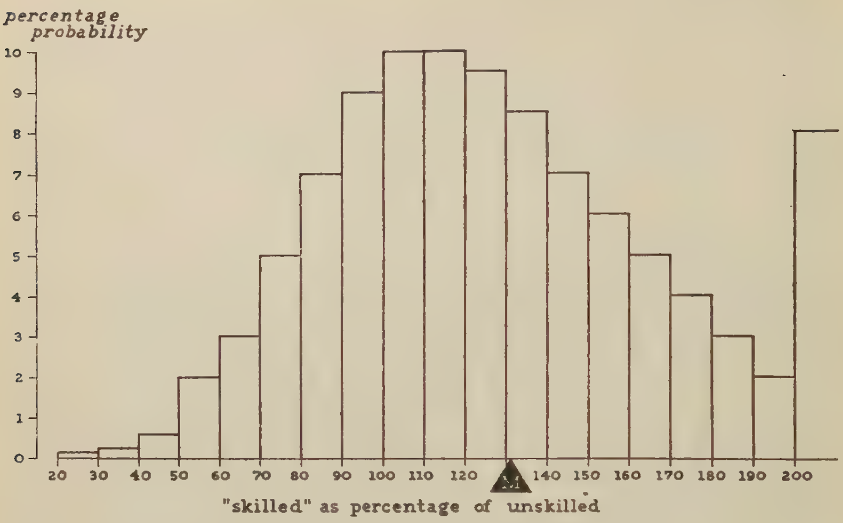
TABLE II

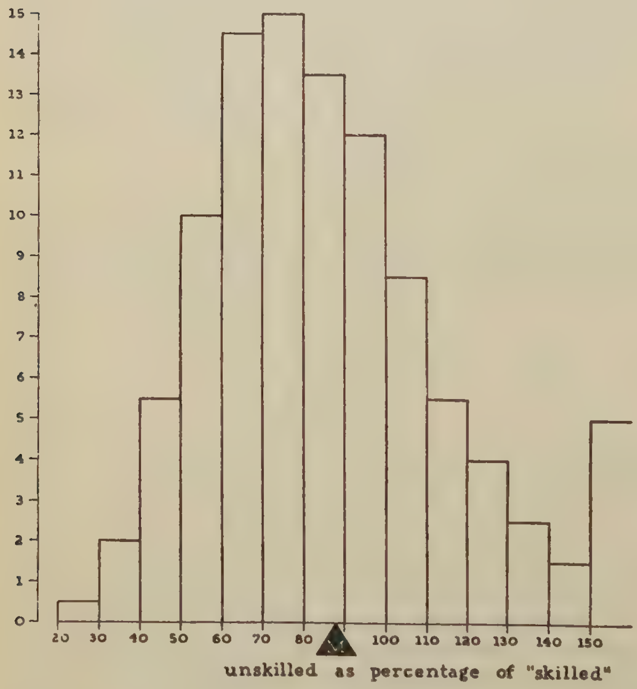
'Averages' of Skill Differentials

	Mean	*Median*	*Mode*	*Geometric Mean*
Skilled as percentage of unskilled	131	123	110[1]	122
Unskilled as percentage of skilled	88	81	71	82

[1] The shape of this distribution makes it difficult to estimate the mode with any precision.

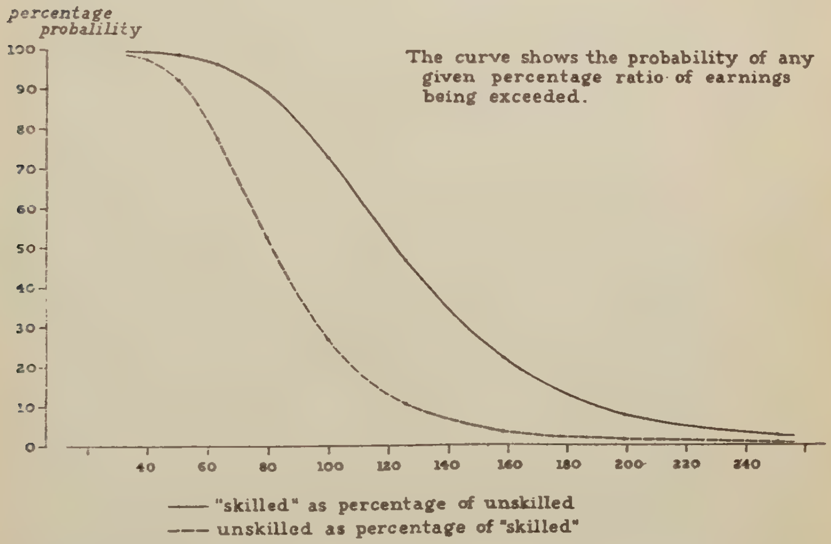
IV. Probability distributions of ratios of earnings





skilled worker will earn *either* at least 55 per cent more than an unskilled worker *or* at least 2 per cent less. Conversely, the quartiles of the lower distribution are 65 and 102, so that the unskilled worker has an even chance of earning *either* at least 35 per cent less than a skilled worker *or* at least 2 per cent more. From the cumulative probability curves (Diagram V) we can read off the odds on or against the percentage ratio being above or below any specified level. For instance, the odds on the ratio being greater than 100 per cent in the upper distribution (or less than 100 per cent in the lower) are slightly under 3 to 1.[1] The unskilled worker need hardly complain: after all, he may be willing to accept infinitely longer odds week after week if, with many millions of others, he ' invests ' in the football pools. The moralist might justly feel as much concern over those who are, possibly, relatively unworthy of their hire as over those who hit the jackpot; for the latter are at least less numerous.

V Cumulative probability of ratios of earnings



The curve shows the probability of any given percentage ratio of earnings being exceeded.

The probability distributions of percentage ratios which we have been considering exhibit a few unusual features. Although the two distributions are complementary, their means are not reciprocals. Thus the skilled have an average differential of 31 per cent above the unskilled, as against an average differential for the unskilled of 12 per cent below the skilled. Not only does each of these figures convey a very different impression of the relative earnings of skilled and unskilled workers, but both differ in turn from the differentials based on averages which we were considering earlier (26 per cent above and 21 per cent below; see Table I). What do these various measures signify ? The latter pair of figures means no more than that the average earnings of the skilled exceed the average earnings of the unskilled by 26 per cent (and conversely that the average for the unskilled is 21 per cent below that for the skilled). It does not mean—although no doubt it is often taken to do so—that ' on average, skilled workers earn 26 per cent more than unskilled ' ; for such a statement should mean only that the average percentage by which a skilled worker's earnings exceed those of an unskilled worker is 26 per cent—which is not true since, as we have seen, this figure is 31 per cent. Indeed, when we are taking all possible pairs (or selecting pairs at random),the average of a series of ratios will in practice always be larger than the corresponding ratio of averages; and the greater the variability of the original distributions the greater the discrepancy between the two measures.

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[1] These odds coincide, of course, with the probability that the difference in earnings between a skilled and an unskilled worker will be greater than zero.

We cannot dismiss the lack of reciprocity between the means of the complementary probability distributions (131 and 88; see Table II) merely as an example of the perverse properties of numbers; for the fact remains that average differentials of ' 31 per cent above ' and ' 12 per cent below ' conjure up very different pictures and, in practice, will be charged with an emotive force which makes them dangerous to handle in isolation. The same applies to the modes—in which case, however, the picture is reversed (the figures being approximately ' 10 per cent above ' and '29 per cent below ' respectively). Of course, the ratios (Table I, column 1) based on the arithmetic means of the original distributions are simply reciprocals of each other and display no such paradox. But reciprocity in itself is not a sufficient reason for preferring the ratio of the arithmetic means to the arithmetic mean of the ratios; for both these measures are, after all, mathematical mongrels.[1] In this context the nearest to a ' thoroughbred ' is the geometric mean; for not only are the geometric means of the probability distributions of percentage ratios reciprocal to each other but they are identical with the ratios of the geometric means of the two original distributions. However, the trouble with geometric means is that it is not easy either to calculate them or to attach a commonsense meaning to them—and their use is therefore hardly to be recommended to negotiators. We are, in effect, faced with the dilemma that the measures which are the least consistent are the most readily comprehensible, and those that are the most easily intelligible have the least convenient algebraic properties.[2]

This dilemma would have serious consequences if we were searching for an ' ideal ' measure. Our present purpose, as we have stated, is rather different; and our exploration of various possible measures of wage differentials has been undertaken primarily to bring out the complexity of the phenomenon that we are dealing with. Because, in practice, differentials are less simple relationships than would appear from collective agreements, the crude measures which are bandied across the negotiating table may be quite inadequate to bear the inferences which are drawn from them—let alone the policy decisions based on the inferences. There is, in fact, no single measure of wage differentials which is appropriate in every case: what is needed is a combination of different measures. When we examine differentials in earnings, we are, from a statistical point of view, comparing two distributions; and we therefore need more than one measure of central tendency as well as some indication of range, dispersion, skewness, and so on. In many cases a knowledge of the original distributions and of their properties would suffice for practical purposes; but where the two distributions are clearly dissimilar it may be useful to undertake a more systematic analysis of their differences of the type we have suggested. Given probability distributions, the most satisfactory measure of central tendency is the median, and the best indicators of dispersion are those based on other order statistics such as quartiles and deciles. For each of these statistics defines specific odds (even chances, 3 to 1, 9 to 1) and, where we are examining percentage ratios and therefore have complementary probability distributions, every order statistic in one distribution is the reciprocal of a corresponding statistic in the other.[1]

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[1] The latter is the more temperamental mongrel of the two ; for the mean of the ratios is a measure which is extremely sensitive to the influence of the tails of whichever of the two original distributions is being used as base—*i.e.* the tail is apt to wag the dog.

[2] A similar dilemma arises with index numbers : thus Fisher's Ideal Index, which alone obeys both ' time- ' and ' factor-reversal ' tests, is seldom used in practice because it is difficult to calculate and to comprehend.

It is obvious that we cannot be sure of obtaining a reliable picture of wage differentials (whether by skill, type of payment, age, region, etc.) without distributions of earnings. In the United States, the *Monthly Labour Review* gives these fairly often in some detail; but in Britain only two official enquiries this century (in 1906 and 1938) have included them—the 1938 distributions being published eleven years after the event, and even then in a highly summary form. Some employers' associations compile distributions of earnings by occupations for their own purposes; but, apart from this, the Ministry of Labour could surely carry out, periodically, comprehensive sample enquiries with this object, at least in the main manufacturing industries. Such an effort, and it would undoubtedly be an effort, would not be wasted. In the last few years a great deal of argumentation, based on the flimsiest of evidence, has been expended on the subject of the (actual or hypothetical) narrowing of the skill differential. The panacea—that of granting uniformly greater increases in wage rates to all skilled grades—certainly has the effect of widening the difference between averages of earnings; but where there is a considerable overlap of skilled and unskilled workers' earnings there is at least *prima facie* evidence of anomalies in the wage structure. For while some degree of overlap can, as we have seen, be justified on grounds of varying intensity of work and so on, it is not impossible that the reward for skill is being largely outweighed by less rational factors. At present, there is almost complete ignorance even of the extent of the overlap in particular industries; and, so long as adequate statistics are lacking, this ignorance will continue to blind those responsible for wage negotiation—not to mention academic economists—to problems which may be far more urgent than the issues that actually emerge.

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[1] The medians are of course reciprocals of each other; the lower quartile of one distribution is the reciprocal of the upper quartile of the other and *vice versa*; and so on.

K. G. J. C. Knowles.
T. P. Hill.

APPENDIX

Method Used to Obtain the Two Probability Distributions

I. *Distribution of absolute differences*

If our two original distributions A and B are grouped into n equal class intervals covering their joint range, then :—

(1) The probability that any individual member α of distribution A selected at random will fall in the ith interval is given by the proportion of the total frequency of A falling in that interval. We shall denote this probability by the expression a_i, and the corresponding probability for any individual member β of distribution B by b_i.

(2) The probability of both α and β falling in the ith interval is the product $a_i\, b_i$. Thus the probability of their both falling in the same (unspecified) interval is $\sum\limits_{i=1}^{i=n} \left(a_i b_i\right)$.

(3) Similarly, the probability of α falling λ intervals lower or higher than β is $\sum\limits_{i=1}^{i=n} \left(a_{i+\lambda}\, b_i\right)$.

(4) Therefore, the probability of α falling in a higher interval than β is $\sum\limits_{\lambda=1}^{\lambda=n} \sum\limits_{i=1}^{i=n} \left(a_{i+\lambda} b_i\right)$.

(5) Similarly, the probability of α falling in an interval not less than k intervals lower or higher than β is $\sum\limits_{\lambda=k+1}^{\lambda=n} \sum\limits_{i=1}^{i=n} \left(a_{i+\lambda} b_i\right)$.

We now have to make the assumption that, if α falls in the same interval as β, the probability of α exceeding β is exactly one-half—which implies that the distribution within each individual interval of both A and B is symmetrical.[1] Although the assumption will be a minor source of error in our derived probability distribution, it is in essence a conventional one. Given this, then :—

(6) The probability that α will be greater than β is equal to the probability

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[1] Or, if not symmetrical, identical.

that α falls in a higher interval than β plus one-half of the probability of their falling in the same interval, *i.e.* : $\sum_{\lambda=1}^{\lambda=n} \sum_{i=1}^{i=n} \left(a_{i+\lambda}\, b_i\right) + \frac{1}{2} \sum_{i=1}^{i=n} \left(a_i\, b_i\right)$.

(7) Similarly, the probability of obtaining a value of $(\alpha-\beta)$, where this difference is measured in units of the class interval, greater than any specified value[1] of k, is $\sum_{\lambda=k+1}^{\lambda=n} \sum_{i=1}^{i=n} \left(a_{i+\lambda}\, b_i\right) + \frac{1}{2} \sum_{i=1}^{i=n} \left(a_{i+k}\, b_i\right)$.

The probabilities obtained from the general expression given in (7) by letting k range from $-n$ to $+n$ are specific values of the cumulative probability function of $(\alpha-\beta)$. By taking the differences between successive values of this function we obtain, of course, the probability distribution itself (*cf.* Diagram II in text). If we wish to obtain the probability of $(\alpha-\beta)$ lying between any specified limits, we can do this by means of graphical interpolation after plotting the cumulative function and joining the points by a smooth curve (*cf.* Diagram III in text). Having obtained the probability distribution (whether uncumulated or cumulated) we naturally need no longer measure $(\alpha-\beta)$ in terms of class intervals but can adjust the scale to the original—or any other appropriate—units.

II. *Distribution of percentage ratios*

In order that we may calculate the probability distributions of the ratio of α to β and of β to α, the data have to be grouped in such a way that the width of each class interval measured in logarithms is a constant (for our calculations, we chose 0.1). Since our only available data were already grouped in distributions having equal absolute intervals, we had to transform these original distributions. The latter were therefore cumulated and plotted, and the resulting curves smoothed; and from each of these cumulative curves the logarithmic distribution is given by the differences between successive frequencies—the frequencies being obtained by graphical interpolation at equal logarithmic intervals.

The probability distribution of the differences between the logarithms of α and β can then be derived by the method described in I.[2] This distribution generates two complementary distributions of ratios, *i.e.* α as a percentage of β and β as a percentage of α (*cf.* Diagram IV in text). For example, the probability of the logarithmic difference being between zero and 0.1 is the probability of the ratio of α to β being between 1 : 1 and 1.26 : 1 (1 and 1.26 being the antilogarithms of zero and 0.1)—*i.e.* of α being between 100 and 126 per cent of β.

By means of interpolation at two points on the cumulative probability curve (*cf.* Diagram V in text), it is possible to obtain the probability of the percentage ratio falling between any specified limits.

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[1] Where the specified value of k is negative, the probability is that of obtaining any negative value of $(\alpha-\beta)$ from k to zero, or any positive value.

[2] Our assumption about the distribution within class intervals is the same as previously: *i.e.* that the distributions within all class intervals of A and B are symmetrical (or, if not symmetrical, identical)—in this case, however, on a logarithmic scale.

JOURNAL *of the* AMERICAN STATISTICAL ASSOCIATION

Volume 49. December 1954 No. 268

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